THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 37, NUMBER 4 APRIL 1965

Analysis of the Flute Head Joint

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(Received 18 May 1964)

The acoustical nature of the head joint of a transverse flute is examined in mathematical detail. The effective length of the flute is altered in the amount by an embouchure correction $\Delta l_e = (c/2\pi f) \arctan\{(c/2\pi)(f/r)/(2f_c^2 - f^2)\}$, where f is the playing frequency, c is the speed of sound in the contained air, r the (17-mm) cork-to-embouchure distance, and f_c a parameter (near 1450 cps) which is essentially the frequency of a Helmholtz resonator made by plugging the bore with a second cork a distance r below the embouchure. This correction is roughly constant at 42 mm over the normal playing range. Effects of cork position, lip position, and embouchure-hole size are discussed in detail, as is the effect of cavity resonances in the player's own mouth. Tuning errors caused by the edgetone-regeneration mechanism are shown to be correctible by a head-joint taper perturbation. Three designs are analyzed, which produce a flattening at low frequencies, zero effect near 600 cps, and a maximum sharpening near 1000 cps. Above, this the correction returns smoothly to zero at the upper playing limit near 2000 cps. In the high register, the correction is produced jointly by the taper and the vent holes. The whole analysis is summarized by means of a detailed comparison of the Boehm design of flute with the older cylindrical-head, taper-bore model and with the purely cylindrical flute.

INTRODUCTION

A. Nature and Scope of the Discussion

The flute has for many years been the object of scientific interest, not only because of its important role in music, but also because of an external simplicity that suggests that it is susceptible to straightforward analysis.[1–5] As is the case with all musical instruments, however, the close constructional tolerances required by the musician combine with the often intangible phenomena of physics, physiology, and psychoacoustics to transform this initial simplicity of the flute into a subtly interacting complexity.

The present report sets forth a mathematical analysis of the flute's head joint as it interacts with the player, together with a brief discussion of the related musical implications of various phenomena.[6]

While the tone color of a flute is determined in part by the same parameters that control its intonation, it is not possible to analyze tone color meaningfully unless resonance properties of the complete instrument are known over the whole spectral domain. Required

[1] A well-known classic on the subject of flutes is T. Boehm, *The Flue and Flute-Playing*, translated and annotated by D. C. Miller (Rudall, Carte & Co., London, 1922).

[2] The history of the flute as a musical instrument is well summarized in Anthony Baines, *Woodwind Instruments and Their History* (W. W. Norton & Co., Inc., New York, 1963), rev. ed., Chaps. 2, 7; see also pp. 248–252, 316–324.

[3] A good general account of the earlier scientific history of the flute is to be found in R. S. Rockstro, *A Treatise on the Construction, the History and the Practice of the Flute* (Rudall, Carte & Co., London, 1928), rev. ed., see especially Chap. 3.

[4] A considerable body of analytical work on the flute is described in (a) H. Bouasse, *Instruments à vent* (Librairie Delagrave, Paris, 1929, 1930), Vols. 1, 2; (b) H. Bouasse, *Tuyaux et résonateurs* (Librairie Delagrave, Paris, 1929). It is unfortunate that no bibliographical information whatever appears in these works. However, most of the references to studies made in the 18th and 19th centuries may be identified in the extensive reference list that appears at the beginning of Rockstro's book (Ref. 3).

[5] Page references to specific topics in these books are given under the author's names at various places in the remainder of this report.

[6] A. H. Benade and J. W. French, J. Acoust. Soc. Am. 34, 716(A) (1962).

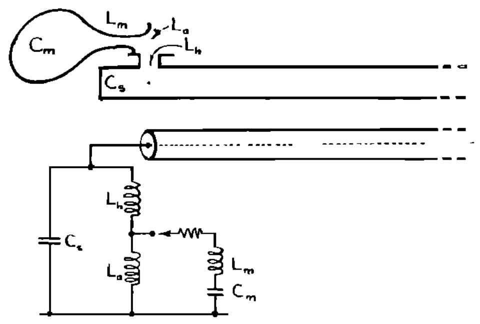

FIG. 1. Schematic diagram of the flute embouchure-hole system, as it is coupled to the player's mouth cavity. The electrical circuit impedance analog is also shown.

in addition is a knowledge of the detailed nature of the regeneration mechanism that sustains the oscillations. Since this paper deals only with the head joint, it would be premature to attempt here a discussion of tone color.

Several informal experiments are described that may be performed by any flautist on his own instrument, to call attention to important or unfamiliar aspects of the problem at hand. They are not presented as the major basis upon which the argument is founded.

It should be emphasized that it is a matter of considerable subtlety to make a meaningful comparison between theoretical calculations of the properties of a flute with any of the various semiempirical recipes that are used in the practical design of flutes. For example, Boehm[7] calculates the position of the nth hole above middle C upon the basis of the following formula: $L_n=[(f_C/f_n)L_C-51.5]$ mm, with $L_C=670$ mm. Here L_n is the distance between the cork and the tone hole for this note and f_n is its frequency. L_C is what Boehm terms the "theoretical" length for the note middle C, and f_C is its frequency. *Note:* L_C is *not* the half-wavelength for this reference frequency, but rather a quantity that involves an integration of the wave velocity over the closed-hole portion of the bore and an open-hole end correction at the lower end of the playing length, as well as contributions from the embouchure-cavity system, the edgetone-regenerative system, and the head-joint taper. Since the present paper deals only with the head joint, comparison between theory and practice is premature.

B. Summary and Outline

Section I is devoted to a study of the boundary condition that is imposed on the upper end of the flute by the nature of the embouchure hole and the cavity that exists between this hole and the cork. As a first approximation, the effect of the player's mouth and throat on the flute's resonance frequencies is ignored. It is shown that, for normal playing on a modern Boehm-design flute, this boundary condition gives rise to an end correction that is very nearly constant with frequency over the whole playing range. The magnitude of this correction (as measured from the center of the embouchure hole) is close to 42 mm. The effect of varying the amount by which the embouchure hole is covered by the lips, and of moving the cork, is also computed.

Section II describes the nature of the alteration that is brought about by the player's mouth cavity, and estimates are made of the magnitude of the resultant tuning changes. It is shown that this effect is musically small, but not negligible, being of the order of 5 or 10 cents. The flute's tuning is relatively unaffected by the presence of the cavity for notes whose frequency is well below that of the mouth resonance. There is an abrupt but essentially constant shortening of the effective length of the flute when the playing frequency rises above a certain frequency that lies just below the mouth-cavity resonance frequency.

Section III gives a phenomenological account of the nature of the edgetone-regeneration mechanism that is used to excite the oscillations of the flute's air column. This account is used chiefly to show the need for a frequency-dependent length correction that is to be superposed on the correction that arises in the neighborhood of the embouchure hole.

Section IV describes two methods by which a frequency dependent correction of the proper sort may be obtained, one having its practical realization in the tapered head joint originated by Boehm and the other in the cylindrical head that was commonly used with conical bore flutes, and is still met with in piccolos. Detailed calculations are presented of the effects produced by several typical head joints, along with an account of the qualitative way in which design changes affect the resonances of the flute. A particularly interesting result of the analysis is the elucidation of the manner in which the needed sharpening effect produced by a head-joint constriction ceases at a frequency (near 1000 cps) that is close to the one at which the sharpening effect of the high-register vent holes begins.

Section V is devoted to a musically oriented recapitulation of the earlier sections, the discussion being cast in the form of a comparison of three major types of head joint. Systematic procedures for adjusting the intonation of two of these are described also.

I. EMBOUCHURE HOLE AND HEAD CAVITY

The upper end of the flute head joint may initially be thought of as providing a terminating admittance for an essentially cylindrical tube that is open at its lower end. This termination consists of the embouchure-hole admittance shunted by the admittance of the small cavity that exists between the hole and an adjustable cork plug. The admittance of the hole is made complicated not only by the mechanical shape of the hole

[7] Ref. 1, pp. 34–37.

through the riser and lip plate of the flute itself, but also by the partial shading of the hole by the player's lips. A further complication is brought in by the fact that the player's mouth and throat cavities are coupled to the flute by way of the aperture between his lips. Figure 1 gives a schematic indication of the nature of the system, along with its electrical-impedance analog. For the purposes of analysis, it is fortunate that the opening to free air between the lip plate of the flute and the player's mouth opening is large enough very nearly to decouple the flute from the mouth cavity resonances. The effects of these resonances can therefore be ignored initially, and afterward brought in as a small correction, as is shown in Sec. II.

A. Formulation of the Problem

In general, the length correction Δl produced by a noninfinite terminating admittance Y_t may be written[8] as

$$\Delta l = (c/2\pi f)\arctan(Y_0/jY_t). \quad (1)$$

Here, Y_0 is the characteristic admittance of the pipe, c is the phase velocity of sound within it, and f is the frequency at which the value of Δl is to be evaluated.

In the problem at hand, the terminating admittance (located at $x=0$ on a coordinate axis running the length of the bore) is the sum of the embouchure-hole admittance Y_h and that of the hole-to cork cavity Y_c.

$$Y_t = Y_h + Y_c. \quad (2)$$

For all musical purposes, the linear dimensions of the hole and cavity are much smaller than the wavelength, so that the low-frequency approximations to the terminating admittances are perfectly adequate. If the bore has a mean radius a in the neighborhood of the embouchure hole and if the hole-to-cork cavity has the length r (as measured from the center line of the embouchure hole), then the two admittances are as given in Eq. (3), along with an expression for the characteristic admittance Y_0 of the main tube.

$$Y_h = -j(K/2\pi f), \quad K = \text{const}; \quad (3a)$$

$$Y_c = +j(\pi a^2 r/\rho c^2)(2\pi f); \quad (3b)$$

$$Y_0 = (\pi a^2/\rho c); \quad (3c)$$

In this approximation, Y_t becomes

$$Y_t = -j(\pi a^2/\rho c)[(K\rho c/\pi a^2)/(2\pi f) - (2\pi f r/c)]. \quad (4)$$

Because it is not susceptible to direct measurement, nor to calculation from the linear dimensions of the hole, the aperture constant K is best expressed in terms of the observable natural frequency belonging to a Helmholtz resonator that is made out of the flute by adding a second cork to the head joint. If this new cork is placed the same distance r *below* the embouchure as the original cork is *above* it, the algebra becomes particularly convenient. This new resonator runs at such a natural frequency as to make the actual hole admittance equal in magnitude to the calculable admittance $2Y_c$ of the two "paralleled" hole-to-cork cavities.

At the Helmholtz resonator's natural frequency f_r therefore, the following relation holds:

$$K = (\pi a^2/\rho c)(2\pi f_r)^2. \quad (5)$$

With this expression for K, Eq. (4) can then be used with the defining Eq. (1) to give an explicit formula for the flute embouchure length correction Δl_e as follows:

$$\Delta l_e = (c/2\pi f)\arctan\left[\frac{(c/2\pi)(f/r)}{2f_r^2 - f^2}\right]. \quad (6)$$

At frequencies for which f^2 rises past the value $2f_r^2$, the argument of the arctangent first becomes singular and then changes sign, and as a result Δl_e also changes sign. On flutes, this phenomenon takes place at a critical frequency near 2000 cps, which is at the top of the instrument's normal range. At the critical frequency, Δl_c is given by

$$\Delta l_{crit} = \pm\tfrac{1}{4}(1/\sqrt{2})(c/f_r). \quad (7)$$

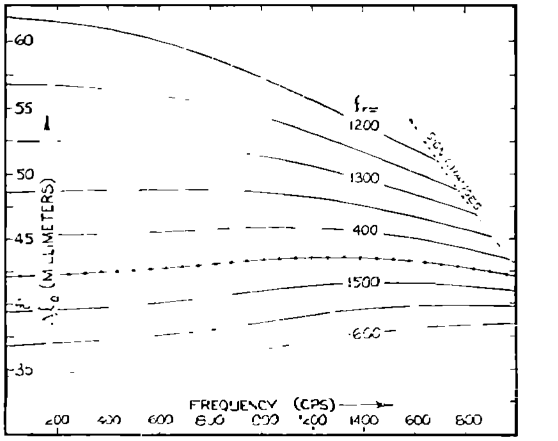

FIG. 2. End correction produced at the embouchure hole, as computed from Eq. (6). The beaded curve shows the behavior when the player's lips are in normal position. Variable f_r. $r=17$ mm.

B. Results and Discussion

Figure 2 shows the dependence of the quantity Δl_e on the playing frequency for various values of the reference Helmholtz resonator frequency f_r. These are calculated for the particular value $r=17$ mm, which is usual on Boehm flutes. Most flutes are built to play well in tune when the player so arranges his lips that f_r is about 1450 cps (the curve for this value is emphasized by being drawn with a beaded line). It is, however, possible to play with the embouchure hole covered

[8] A definition and brief discussion of the meaning of the length correction as used here is given in A. H. Benade, J. Acoust. Soc. Am. 32, 1591 [1608](1960), Secs. IV A and V A, see Eq. (30). In this reference, the correction is expressed in terms of the impedances Z_t and Z_0 rather than the corresponding admittances.

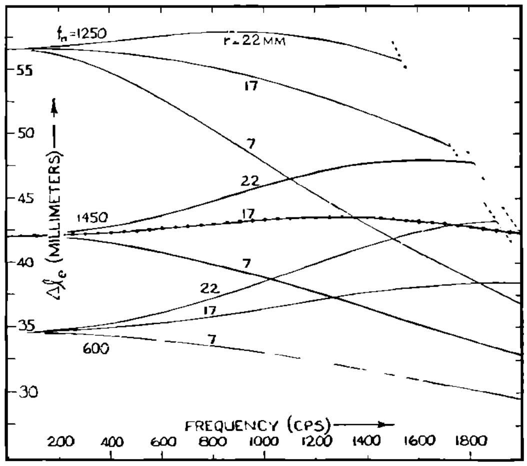

FIG. 3. Variation of the embouchure end correction Δl_e produced by moving the head joint cork as calculated from Eq. (8). The beaded curve shows the normal behavior for a Boehm flute.

enough to lower f_r as far as 1200 cps or uncovered enough to raise it to nearly 1600 cps.[9]

An interesting feature of the length correction (as described above) is its almost perfect constancy over the musical frequency range, if the player's lip is in normal position. The musician can, of course, alter the frequency of his instrument quite considerably by rolling the flute in or out on his lip, which serves to alter the value of f_r.

For a fixed lip position, f_r^2 varies inversely as the volume of the Helmholtz resonator cavity, which is in turn directly proportional to r. Let f_{17} be the value of f_r that is obtained when $r=17$ mm with the player's lip in some definite position: this same lip position will then give a resonator frequency $f_r=(17/r)^{\frac{1}{2}}f_{17}$ when a new value of r is used. Equation (6) may therefore be written in the following form

$$\Delta l_e=(c/2\pi f)\arctan\left[\frac{(c/2\pi)(f/r)}{2(17/r)f_{17}^2-f^2}\right]. \qquad (8)$$

Figure 3 shows the manner in which the frequency dependence of varies with r for three different values of f_{17}. Once again, the curve for normal playing conditions ($r=17$ mm; $f_{17}=1450$ cps) is emphasized by being shown as a beaded line. The curves in Fig. 3 may be summarized as follows: At the bottom of the flute's range, even drastic alterations in the value of r do very little to alter the frequency of the played note.[10] If higher notes are considered, increasing the value of r from 17 mm flattens the frequency, while decreasing r sharpens it. This can be put in another way, which is more useful in the musical context: Everything else being equal, Δl_e remains essentially constant with frequency if r is near 17 mm. Increasing r has the effect of shrinking the basic musical intervals that relate the frequencies of the various normal vibrational modes of the instruments. Decreasing r has the converse effect of widening the intervals. The order of magnitude of the effect is small: about 2 cents per octave change is produced for each millimeter that r is altered.

In conclusion, it is to be emphasized that the dominant linear dimensions that control expression for Δl_e are the bore diameter, embouchure-hole "diameter," the height of the rise, and the cork-to-hole distance. All of these tend to stay in rough proportion to one another as the flute's dimensions are decreased from those of an alto flute through the concert flute, down to the piccolo. These dimensions vary as the square root of the *length* of the instrument, rather than as the first power,[11] and as a result each size of instrument has a value of f_r that is consistent with its general frequency level. The quantitative considerations discussed here apply accurately only to the concert flute but very similar qualitative conclusions hold for the other instruments of the flute family.

II. EFFECTS DUE TO PLAYER'S MOUTH AND THROAT

Since the analysis of the flute's embouchure system itself as a passive device has been essentially completed in the foregoing section, the next step is to inquire into the effect of perturbations produced by the player's mouth and throat on the nature of the terminating impedance of the flute tube.

A. Formulation

Addition of the player's mouth-cavity system to the upper-end termination of the flute has the effect of shunting a *part* of the embouchure hole by a device whose admittance function has resonances and antiresonances. Near a resonance of this added system, the flute will behave as though it is loosely coupled to a Helmholtz resonator. Near the antiresonances of the player's system, the flute termination reverts to the one that has already been analyzed. Attention need therefore be focused only on the perturbations produced by a single-degree-of-freedom system that is coupled to the flute.

In order to preserve as much as possible of the earlier analysis, it is worthwhile to construct a modified expression for the reference frequency f_r. Straightforward but tedious algebraic manipulation shows that at a playing frequency f the new f_r is given by the following formula (in the notation of Fig. 1)

$$f_r'=(1/2\pi)[2C_s(L_h+BL_0)]^{-\frac{1}{2}}, \qquad (9)$$

[9] When f_r is 1450 cps, the value of Δl_{elt} implied by Eq. (7) is close to 42 mm.

[10] In this part of the flute's range, the effective tube length is around 500 mm, so that a change of 1 mm in the effective length corresponds to a frequency change of about 3 cents.

[11] A. H. Benade, J. Acoust. Soc. Am. 31, 137–146 (1959), Sec. VII-B.

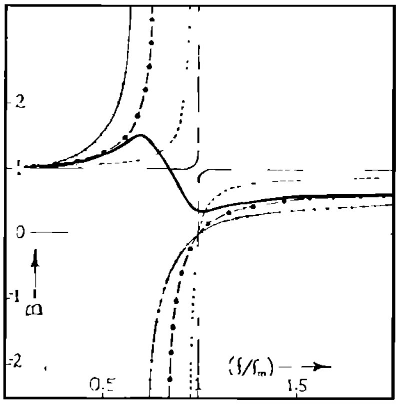

FIG. 4. Frequency dependence of the player's mouth-cavity perturbation coefficient B. The heavy line shows the typical behavior in practice. $\delta=2.0$. $\delta=1.5$. $\delta=1.1$. $\delta=1.0$. $\delta=1.5$, $Q=3$.

where

$$B=\text{real part}\left\{\frac{[(f/f_m)^2-1]-j[(f/f_m)/Q_m]}{[(f/f_m)^2\delta-1]-j[(f/f_m)/Q_m]}\right\}, \quad (10)$$

and

$$\delta=[1+(L_0/L_m)]. \quad (11)$$

The parameter f_r' is still to be interpreted as the resonant frequency of the doubled hole-to-cork cavity, but now the admittance of the aperture is altered by the near presence of a resonator whose natural frequency is f_m and whose quality factor is Q_m. Because of this alteration, f_r' is not a true constant even when the player keeps a fixed lip position on his flute, and it varies[12] with playing frequency by way of the disturbance coefficient B.

B. Summary of Mathematical Results

The qualitative nature of the phenomena brought into being by the player's mouth cavity may conveniently be approached by way of a study of the frequency dependence of the factor B. Equation (9) shows that the value of f_r' to be used in Eq. (6) shrinks as B grows, and as a result the flute is flattened by an increase in B. Figure 4 shows the dependence of B on the frequency for several values of the parameter δ: four of the curves are plotted under the assumption of infinite mouth resonator Q, while the fifth shows a more realistic situation where both δ and Q are chosen to approximate values appropriate to an actual flute. The behavior of all the curves may be summarized rather easily as follows: (a) At low playing frequencies, B approaches unity, so that the disturbance to the flute goes to zero. (b) At high frequencies, B approaches $(1/\delta)$, so that the flute is sharpened slightly. (c) The transition between the two limiting values occurs in the neighborhood of $f=f_m/\sqrt{\delta}$. (d) If the damping is low, B vanishes when $f\cong f_m$. The dominant musical implication of this calculation is the prediction of a "jog" in the intonation, such that the flute is slightly flatter when played at frequencies below $f_m/\sqrt{\delta}$ than it is when played above, and that in the immediate neighborhood of this transition frequency there may be large discontinuities in the tuning[13] of adjacent notes.

C. Estimation of Parameters and Magnitude of the Effect

It is possible to make reasonable estimates of the magnitude of the tuning jog produced by the player's mouth and also the frequency region in which it takes place. Furthermore, it is possible to show the existence of the effect by simple experiments that are accessible to any flute player.

When the flute is in normal playing position, direct inspection shows that the player's mouth aperture is considerably smaller than is the open passage from the top of the lip plate into the open air. This shows that the ratio (L_0/L_m) of the impedances of these two apertures is less than unity, and that therefore $1\lesssim\delta\lesssim2$. It is reasonable to take $\delta=1.5$ as a good approximation to the actual situation. Upon this basis, it is possible to estimate the magnitude of the frequency shift as being in the neighborhood of 5% or 10% of a semitone, an amount that is relevant to the design and playing of flutes. When playing a flute, the musician's mouth and throat are in roughly the same position as the one he uses for pronouncing vowel sounds like "ah," "aw," or "oh," except that his larynx is pulled out of the way so as to leave a relatively more free passage to the lungs and his lips are puckered more. It is plausible to expect that value of the resonance frequency that is appropriate for use in Eq. (9) should be somewhat lower than the frequency of the lowest formant for the kind of vowel described above, a frequency that lies[14] in the range of 500-600 cps. The second formant for these vowels falls in the range of 1200-1800 cps, and so are far enough removed as to cause no complications, while the third formant is higher yet. If, now, the musician keeps his lips steady, but moves his tongue so that he alternates between "pronouncing" his normal playing vowel and one having a drastically different formant structure, then notes that are in the neighborhood of his normal f_m will have their tuning shifted somewhat when he uses the new vowel. On the other hand, notes whose frequencies are well away from the normal f_m are expected to be unaffected by these changes, because the factor B is essentially constant above and below f_m. A good alternative vowel to use

[12] Attention is called to the fact that, if the player closes his lips, the "inductance" at the flute's embouchure hole is $L=L_h+L_0$, while open lips have the effect of multiplying L_0 by the frequency-dependent factor B [see Eq. (9)]. In the neighborhood of a mouth-cavity antiresonance, a new coefficient must be found to take the place of B: it has the property of reducing to unity at the antiresonance frequency and so making the system behave as though the lips were closed.

[13] Tuning errors arising in this way cannot be corrected by a simple alteration of finger-hole positions or sizes without introducing identical errors (with sign reversed) in other octaves where the same holes are in use.

[14] W. A. Van Bergeijk, J. R. Pierce, and E. E. David, *Waves and the Ear* (Doubleday & Co., Inc., Garden City, N. Y., 1960), Table II, p. 188.

in such experiments is the one found in the word *heed*, for which the first formant is as low as 270 cps and the second one up to 2290 cps.

Playing each note of the chromatic octave that descends toward middle C while the tongue shifts back and forth between its "ee" and "oh" positions shows very little effect above the note A#, but that particularly in the neighborhood of G# and A (near 430 cps) there is an appreciable shift in playing frequency associated with the change of vowel. The sign of the shift is consistent with the assumption that f_m has been moved from above to below the played frequency. That is to say that the played note associated with the high frequency formant "oh" is flatter than that belonging to the low-frequency formant "ee." A similar sensitivity to the vowel change is to be expected in the neighborhood of the formant frequency of the alternative vowel and this is indeed sometimes observable in the neighborhood of C (280 cps), except that here the frequency change is much less prominent and can have either sign, depending on the exact way in which the "ee" is pronounced.

III. EFFECTS DUE TO EDGETONE MECHANISM

Every flautist is very much aware of the sensitivity of the frequency to the velocity of the air jet that he projects across the embouchure hole. The flute has a tendency to go flat when played pianissimo and sharp when blown at maximum loudness. As a result, the intonation must be corrected during changes of dynamic level by changing of lip positions, by rolling the instrument about its axis, or by nodding the head. Small but rapid changes are also made by exploiting the coupling between the flute and the highly mobile mouth cavity of the musician, as described in the preceding section. Since the wind-velocity dependence of the frequency differs in the various parts of the flute's scale, it is apparent that an instrument that is built with satisfactory intonation in one register, and at one dynamic level, could be off pitch when played at the same loudness in another register. Before analyzing the design of actual flutes as it is affected by the edgetone regenerative mechanism that sustains their oscillations, it will be necessary to describe the major phenomena.

A. Phenomenology of Edgetone Regeneration

There is an enormous literature on edgetone oscillations, both experimental and theoretical, a certain amount of which is controversial. The present summary is based on an extensive discussion by Bouasse[15,16] from the point of view of its musical implications. Fortunately, everything that will be required for present purposes can easily be checked with the simplest

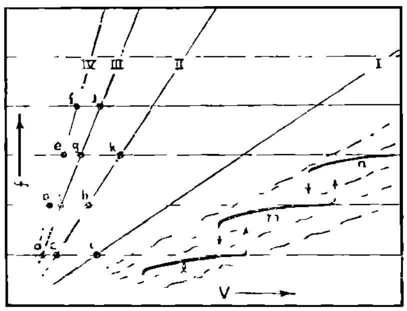

FIG. 5. Schematic diagram showing the edgetones produced by a jet-edge system that is under the influence of a cylindrical pipe.[3] V is the air-jet velocity, and the horizontal lines indicate the natural frequencies of the pipe. The points a, b, c, etc., represent the *régime buccale* frequencies, while the heavy lines l, m, n represent the *régime normale* frequencies that are used in music.

of experimental equipment, and theoretical explanations of the various phenomena are not needed.

There are two distinct classes of regenerative behavior associated with a jet-edge system that is coupled to a resonator, one of which appears at low blowing velocities and the other at higher velocities. Bouasse refers to the first of these as the *régime buccale*, and the second as the *régime normale*: it is the latter that is of primary interest to musicians, while the former is perhaps easier to analyze mathematically.[17] Consider first a simple air jet directed against a knife edge located at a fixed distance h. For any given air velocity V, one or more whistling sounds will be produced: these are not in general harmonically related, but their frequencies f_j all vary with V, h, and an ordering parameter j in accordance with an equation[18] of the following form:

$$f_j=\alpha j(V-\beta)[(1/h)-\gamma]. \tag{12}$$

Here α, β, and γ are constants and $j=1, 2.3, 3.8, 5.4, \cdots$. The lines marked I, II, III, and IV on Fig. 5 represent these edgetone frequencies in a schematic fashion for increasing values of the parameter j. When the air velocity is reasonably large, a rushing sound can be heard in addition to the whistling. The listener can sometimes associate a certain crudely defined feeling of pitch to this rushing noise, a pitch that lies considerably lower than that of the whistlings. The partially shaded region that slopes up from left to right in Fig. 5 suggests this impression of pitch as it is related to the air velocity.

So far, the description has been confined to the case of an isolated jet-edge system. If, on the other hand, the jet and its associated edge are brought within the "domain of influence" of a resonator—i.e., within a distance about equal to the transverse dimension of an aperture of the resonator—a new set of phenomena comes into being. Because of its relevance to the flute, it is convenient here to restrict attention to a resonator whose natural frequencies form a complete harmonic series: the set of horizontal lines drawn across the

[15] Ref. 4(a), Vol. 1, Chap. 4, Sec. 96 103; Vol. 2, Chap. 9, Secs. 143, 156–160.
[16] Ref. 4(b), Chap. 3, Secs. 52–160.

[17] In any event, dimensional analysis gives the same results for both régimes.
[18] G. B. Brown, Proc. Phys. Soc. (London) 49, 493–507 (1937).

face of Fig. 5 represent these evenly spaced natural frequencies of the resonator. If, now, the air velocity is gradually increased from zero, essentially no sound is heard except at values of V corresponding to the *intersections* of the sloping edgetone frequency lines with the horizontal lines that represent the normal-mode frequencies of the resonator. If V is steadily increased from zero, the sounds produced in order will (for the example at hand) be those labelled by the letters a, b, c, $\cdots$, so that the impression of a random sequence[19] of notes is given to the listener. This set of discrete frequencies is the manifestation of what Bouasse calls the *régime buccale* in the interaction of a jet with its resonator. In the *régime buccale*, the pipe does not speak except when the frequency of one of its modes of vibration is equal to one of the edgetone frequencies of the isolated system. It is to be emphasized that these sounds come and go abruptly, as V is varied, and their frequencies are not "pulled" up or down by small variations in V. Furthermore, these sounds are not harmonically related to each other unless the resonator has one of the particular shapes that give rise to harmonically related eigenfrequencies. The *régime buccale* is not normally used in musical instruments.

As the air velocity is further increased, to the point where the frequencies contained in the rushing noise become comparable with the first resonance frequency of the pipe, a new and generally stronger sound is heard, which is initially at a lower frequency than that of the lowest resonance of the pipe. As V increases, this frequency rises toward that of the pipe, as indicated by the heavy line in Fig. 5. The shape of this curve has many features that are reminiscent of the behavior of a beating reed that is coupled to a resonator: Just as in the case of a mechanical reed, the sound produced is *lower* in frequency than the correctly calculated resonance frequency of the pipe itself, rising toward it asymptotically until a blowing velocity is reached for which the system overblows to a frequency near that of the next higher resonance of the pipe. This overblowing phenomenon is connected with the change in sign of the net reactance of pipe's input impedance as the frequency is moved through resonance. If there is positive feedback (and so the possibility of regeneration) on one side of the resonance frequency, then there is of necessity negative feedback on the other side of resonance. The heavy lines marked l, m, n, in Fig. 5 represent schematically the frequencies that can be excited by the air jet in the region of the rushing noise. The vertical dotted arrows connecting these curves represent the transitions between the various frequencies; the upward arrows show transitions observed when V is increasing during the course of the experiment, while the downward arrows represent the transitions occurring when V is decreased steadily from a high value. The presence of hysteresis in these transition curves is very characteristic, and has great musical importance, since it permits the musician to play with a wide variation of dynamic level on either one of a pair of registers, while retaining the possibility of fast transition back and forth between them. The set of regeneration frequencies that has just been described[21] is the one that is a manifestation of the *régime normale* of the jet resonator interaction. Bouasse's use of the adjective *normale* for this type of interaction is a reflection of the musical importance of the class of oscillation and does not carry any implication of a hierarchical relation of the two régimes in their interest to physicists.

To recapitulate, in the *régime normale*, the wind velocity is high enough to ensure that all the frequencies associated with the isolated jet-edge system, as described in Eq. (12), are very much higher than the pipe-resonance frequency that is being excited. Each pipe resonance has associated with it a regeneration frequency that varies slightly with V but that keeps its type over a considerable range of V. This is in contrast with the sounds produced in the *régime buccale*, which come and go abruptly, as V is varied. The transitions between the different *régime-normal* frequencies show a marked hysteresis, while such behavior is not characteristic[21] of the *régime buccale*.

B. Musical Consequences

The curves belonging to the *régime normale* in Fig. 5 were plotted to show frequency as a function of wind velocity. Before many conclusions can be drawn concerning the musical design of a flute, it is necessary to deduce the relation between the playing frequency and the pipe's own normal-mode frequencies for fixed wind velocity. The qualitative nature of the new relation can be obtained upon the basis of the following observations: Figure 5 shows that for a value of V that permits either of two playing frequencies, the upper of these is considerably flatter (relative to the pipe's own natural frequency) than is the case for the lower note. This implies that if a pipe is steadily shortened while being blown at constant velocity, the played frequency rises somewhat less rapidly than does the natural frequency of the pipe. There is also a consequence of the fact that the Q (and so also the input admittance) of a pipe driven at the frequency of its nth normal mode is

[19] The initial "cough" or "chiff" of a flue organ pipe arises from the rapid traversal of this sequence of sounds as the air pressure builds up in the pipe foot.

[20] S. A. Elder, J. Acoust. Soc. Am. 35, 1901(A) (1963).

[21] All of the phenomena described above can be demonstrated with a flute (the small whistling noises that often accompany normal playing are *régime buccale* sounds produced at the low-speed edges of the air jet), however, the head joint alone is considerably easier to manage. A piccolo head joint provides an ideal device with which to demonstrate: the lowest *régime buccale* note is particularly loud, and the corresponding *régime normale* note starts about two semitones flat and rises to the frequency of the *buccale* note only when it is played double forte. Bouasse's use of the adjective *buccale* does not refer to any connection with the player's mouth, but rather arises from the organ builder's terminology, which calls the jet-edge system the pipe's "mouth."

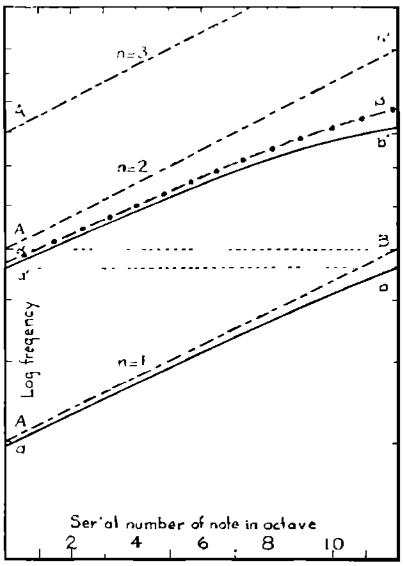

FIG. 6. Diagram of the frequencies of various notes of the simplified flute (actual and desired). The line AB is the desired curve for the low register (based on the first pipe mode). A'B' is the continuation of this same line into the second playing register. The lines ab and a'b' show schematically the flattening produced by the regeneration mechanism. The line $\alpha\beta$ shows the error remaining in the second register if the tone holes are moved to make ab coincide with AB.

equal to the admittance at this same frequency of a pipe of the same diameter whose length is $(1/n)$th of the first one, and which therefore is being driven at its own *first* resonance frequency.[22] Since the jet-edge system at the top of the flute is "informed" about the nature of the pipe only by way of the pipe's input admittance, the conclusion is drawn that the instrument plays flat an equal amount whether a given note is played as the lowest mode of a short pipe or as the second mode of a long pipe[23] (insofar as the regeneration system is concerned). In any event, the regeneration mechanism leads to a progressive flattening as the playing frequency rises.

Figure 6 shows the relation between the various frequencies of interest as a function of the serial number of the note in the chromatic scale that starts upward from middle C. In order to simplify the display, the ordinate is logarithmic so that the frequencies of the desired even-tempered scale form a straight-line sequence. The dashed lines AB and A'B' represent two such sequences of frequencies. Consider a flute constructed in such a manner that the resonance frequencies of its $n=1$ vibrational mode fall along the line AB, while the $n=2$ resonances similarly fall along the line A'B', complete account being taken of all the end corrections and other perturbations to the passive system. If such a flute were to be played with constant wind velocity, the scale of notes would sound flat, with frequencies of the sort suggested schematically in Fig. 6 by the solid line ab for the notes in the low register, and by the line a'b' in the second register. Since the *vertical* distance between the lines AB and A'B' represents a 2:1 frequency ratio of the logarithmic frequency scale, it is clear not only that all the played notes are flat, but also that there is a shrinking of the "octave" relation between notes played with the same fingering in the low and second register. Relocation of the finger holes so as to bring one or the other register into tune is perfectly possible, but such an operation cannot correct both registers. The curve marked $\alpha\beta$ in Fig. 6 shows the behavior of the second register if the flute were modified to bring the first register into strict tune: the vertical distance between this curve and that marked A'B' is a measure of the discrepancy left in the tuning of the second register.

[22] This remark holds for the case where the dominant term in the damping coefficient is associated with thermal and viscous losses to the walls of the pipe, as is the case for flutes. See Ref. 11, VII-A, for a discussion of the analogous relation of Q to mode number for reed instruments.

[23] The relation between the second and third playing registers of a flute is similar to the one just described.

C. Correction of the Discrepancies

The preceeding discussion implies that, if a flute is to play reasonably well in tune without the need for constant lipping of the notes by the instrumentalist, an additional frequency-dependent length correction must be provided, whose general behavior is such as to compensate the flattening that arises in the regeneration process. Instrument makers have long known empirically that a small alteration in the bore cross section will alter the frequency of all the notes that use the perturbed bore as a part of their resonant length. The detailed nature of the alteration depends on the position of the perturbation relative to the nodes of the standing wave, and therefore (in the problem at hand) upon the value of Δl_c. A neat solution of the octave intonation problem should be considered as one of Theobald Boehm's major contributions[24] to the design of flutes. The old-style cylindroconical flute and the modern piccolo are provided with an analogous method of getting acceptable intonation. These are described in the course of the next section.

IV. HEAD-JOINT PERTURBATION

A. Requirements on the Perturbation

If the flute is to play in tune, the bore perturbation must give rise to a length correction that decreases

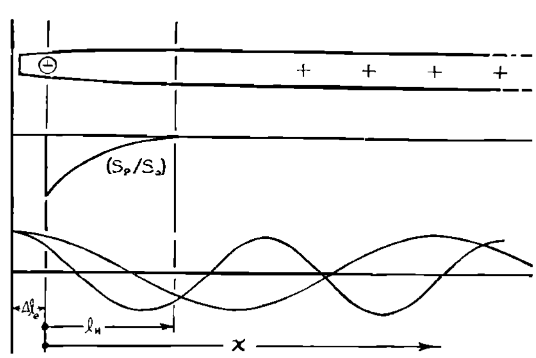

FIG. 7. Diagram showing the head-joint taper perturbation and the cosine weighting functions used in Eq. (13).

[24] Reference 1, Chap. 2.

as the playing frequency goes up. Furthermore, if the scale is to be smooth across the "break" i.e., if the intonation at the upper end of the lower register is to be consistent with that of the lower end of the upper register—the perturbation must lie in a part of the bore that is not removed from the sounding length of the instrument as the fingerholes are successively opened. That is, it must lie in the general region of the head joint, and not lower down the instrument.

If the head joint of the flute has its cross-sectional area modified by a perturbing amount S_p from the normal cross section S_0 of the rest of the instrument, as shown in Fig. 7, the resulting length correction Δl_t associated with a playing frequency f is given[25] by

$$\Delta l_t = -\int_0^{l_H} (S_p/S_0)\cos[(4\pi f/c)(x+\Delta l_e)]dx. \quad (13)$$

Formally, the length correction is the Fourier transform of the perturbation. The necessity of limiting the perturbed part of the bore to a length that is only a fraction of a wavelength for most of the relevant frequencies makes difficult the problem of deducing the desired perturbation (from the *error* that is to be corrected) by means of an inverse transformation. However, the general problem has already been solved empirically by means of two classes of perturbation to the head joint.

B. Suitable Head-Joint Perturbations

Examination of Eq. (13) shows that any perturbation of finite extent that is restricted to lie in the neighborhood of $x=0$ will have an effect whose magnitude is largest at low frequencies, with a sign that depends on the sign of S_p. If in particular the bore is *reduced* near $x=0$, the instrument will be flattened, and in an amount that decreases as the frequency rises. Figure 8 shows the situation for the two major types of flute. A cylinder open at both ends and a cone open at both ends are alike in that their natural frequencies belong to the exact harmonic series. The flute may be based on either one of these shapes, and a reduction of the bore cross section at one end will have the same

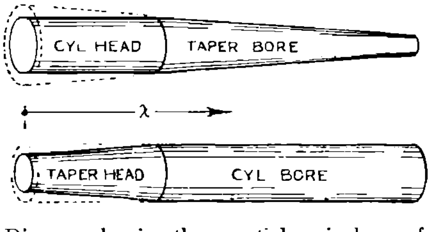

FIG. 8. Diagram showing the essential equivalence of two ways of producing a suitable head-joint perturbation.

[25] Reference 8, p. 1597, Eq. (19). This expression is accurate for a Boehm type cylindrical bore, and reasonably dependable when applied to a conical-bore instrument of normal proportions. Notice that the embouchure correction Δl_e enters this formula explicitly, for reasons that become clear from examination of Fig. 7.

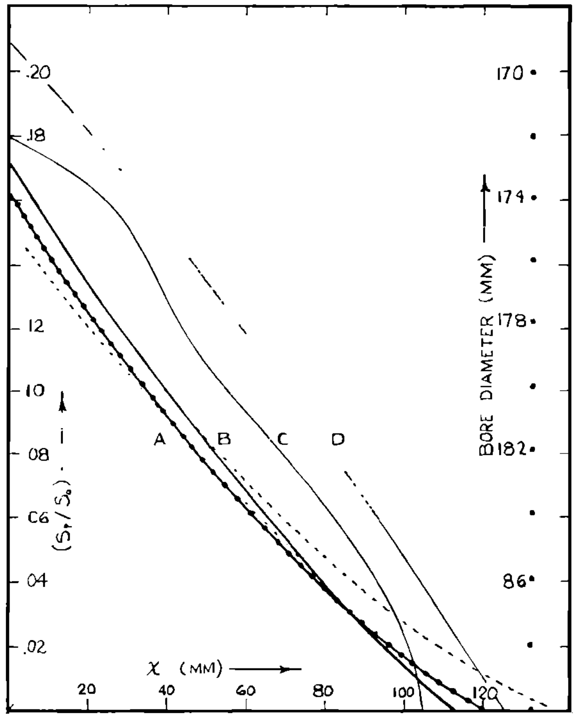

FIG. 9. Experimentally determined fractional area perturbations for various flutes. The beaded line A is the curve for Boehm's own design. Solid curve B is the average for modern flutes. Curve C describes the perturbation for a Rudall Carte flute, and D is that for a conical bore, cylindrical-head flute used by Quantz.

effect in either case. The lower modes are flattened relative to the higher ones, and as a result the basic octave of the instrument is stretched to compensate for tuning errors brought in by the regenerative mechanism or by other effects.

Figure 9 shows some of the perturbations that are actually used in flutes of various designs. The heavy beaded line, marked A, represents the perturbation chosen[26] by Boehm. Curve B is the average of a dozen modern flutes, including instruments made by Powell and both the Haynes brothers. These flutes generally differ less between themselves than the discrepancy shown between curves A and B. The lighter dotted lines show the design used by Selmer (lower) and by Gemeinhardt (upper). All of these perturbations belong to metal instruments that are adapted to the French school of flute playing that is usual in the United States of America. Curve C belongs to a Rudall Carte ebonite flute. This instrument works very well with the "tighter" Germanic style of playing,[27] but seems to waste wind and certainly gives a rather stretched octave when played in the French manner. A conventional Haynes head joint adapted to this flute converts it into a very satisfactory instrument for the American player. The modern Artley flute has a head very similar to the Rudall Carte design, no doubt to preserve the intona-

[26] Reference 1, p. 18.
[27] Reference 3, p. 440, describes a method for obtaining this tone.

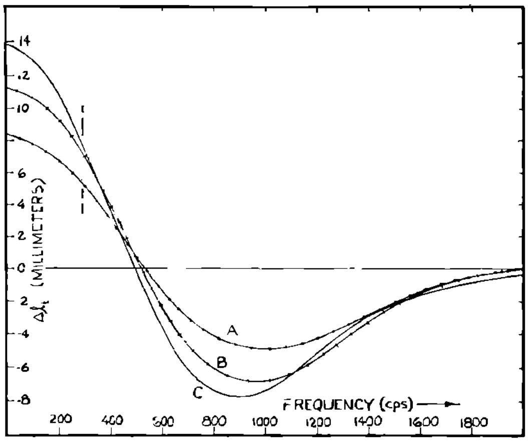

FIG. 10. Frequency variation of length correction Δl_t produced by three head-joint designs. Curve A applies to Boehm's design, B to the Rudall Carte instrument, and C shows the behavior of Quantz' flute. $\Delta l_e = 45$ mm.

tion when the instrument is used by beginners, who tend otherwise to play with a shrunken octave. Curve D represents the perturbation produced by the cylindrical head joint working with a taper-bore instrument. The numbers are based on the dimensions of an instrument made by F. Boie before 1724 and used[28] by J. J. Quantz.

C. Nature and Magnitude of the Effect due to the Perturbation

Numerical integration of Eq. (13) with perturbations appropriate to the Boehm, Rudall Carte, and Quantz designs gives results for Δl_t as shown in Fig. 10. Curve A belongs to the Boehm flute, while B and C belong to the Rudall Carte and Quantz instruments. These were all computed by use of a value of 45 mm for the embouchure-length correction Δl_e that is close to the normal for players of the French school using a Boehm design. The Quantz instrument has a value of Δl_e that is close to 48 mm. This large value arises from the increased bore cross section at the embouchure hole, which lowers the value of f_{17} [as defined just before Eq. (8)] to very nearly 1350 cps. The implication of this increase is made apparent later on in this section.

Several things are immediately obvious. Since the length of the perturbation along the bore is approximately the same for all three designs, the zeros of the length-correction function fall very nearly at the same frequencies for all of them. Furthermore, the magnitude of Δl_t at any frequency is proportional to the total area under the (S_p/S_0) curve for each design of flute and, since the region under each curve of Fig. 9 is roughly triangular, the value of the length correction is approximately proportional to the square of the maximum value of (S_p/S_0). The decrease of Δl_t (and so the progressive sharpening of the flute) with rising frequency over the first two octaves is roughly linear in all three designs. It therefore follows from the preceding sentences that the curves of Fig. 10 have slopes that are proportional to the squared reduction in cross-sectional area at the embouchure hole. For Boehm's design, the flute is effectively shortened by about 2 mm for every 100-cps rise in playing frequency up to about 1000 cps. This corresponds to an octave interval stretching of about 17 cents for notes played with most of the finger holes closed (near the note D), and about twice this for notes played with nearly all the holes open. The Rudall Carte flute and the one used by Quantz have their octaves stretched by amounts which are, respectively, 1.6 and 1.7 times greater than that given for Boehm's instrument.

The positions of finger holes required to give a scale that is continuous across the "break" between the first two registers clearly depends on the design of the head joint. It can be shown that such a hole-positioning scheme can only be carried out if Δl_t decreases in roughly linear fashion for about two octaves.

One further change produced in the tuning of the lower two octaves by the reduction of head-joint cross section is the following: The octave relation is altered when the player covers more or less of the embouchure hole with his lips, not only because of the frequency dependence of the embouchure correction Δl_e, but also because of the dependence of the taper correction on Δl_e. Figure 11 shows the change as computed for Boehm's flute. Here, Δl_t is displayed as a function of frequency for three values of the embouchure correction. The beaded middle curve, for which Δl_e is 45 mm, is close to the normal one, while the others represent extremes that can be obtained in playing. An increase in the value of Δl_e slightly decreases the magnitude of Δl_t at low frequencies, but, more importantly, it gives also a steeper slope to the curve of Δl_t vs f. This has the result of widening the basic octave relation between the lowest two registers, an effect that adds to the small

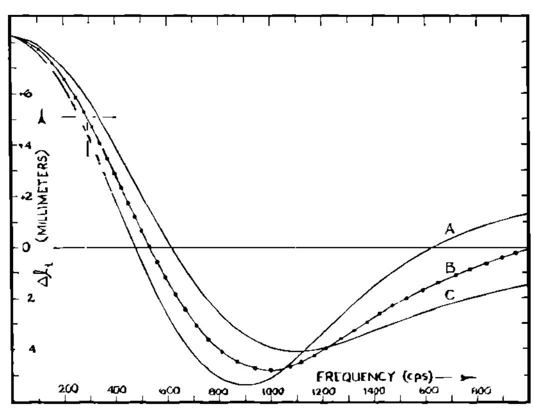

FIG. 11. Effect on the taper correction of varying the embouchure correction Δl_e, calculated for Boehm's head joint. Curve A: $\Delta l_e = 55$ mm. Curve B: $\Delta l_e = 45$ mm. Curve C: $\Delta l_e = 35$ mm.

[28] Reference 3, pp. 227–229. One of the present authors (AHB) has in his possession a 19th century 6-key flute of identical bore design.

amount of widening that is produced directly by the frequency dependence of Δl_e.

The intonation of the third or high register of the flute depends on the head-joint perturbation in a more complicated way than is the case for the lower two registers. Figures 10 and 11 show that, once the frequency is higher than about 1000 cps (which corresponds to the C above the treble staff), Δl_t ceases to sharpen the flute and begins to flatten it progressively over the next octave by increasing from about −5 to about 0 mm (Boehm's flute). This by itself would cause the flute to play somewhat flat, since the regenerative mechanism continues to compress the playing intervals relative to the normal-mode frequency ratios of the pipe. However, in normal playing the opening of vent holes (which are inevitably higher[29] than their naively prescribed positions at the pressure nodes) sharpens the higher normal-mode frequencies, and so makes up the deficit left by the head joint. If the vent hole is a distance ϵ away from the position of the node, then the sharpening of the note can be given in terms of an equivalent shortening of the bore by an amount Δl_v as given[30] by the following equation for the case of the even-numbered higher modes such as those used in playing from E♭ up through G♯:

$$\Delta l_v = \epsilon^2 (b/a)^2 (1/l_{eff}). \qquad (14)$$

Here, b is the radius of the vent hole, l_{eff} its effective length, while a is the radius of the bore, as before.

D. Qualitative Experiments

The qualitative nature of the alterations produced by the head-joint corrections and their relation to the vent hole is easily demonstrated by an analysis of the following simple experiments performed with a flute: If the low D is fingered, and the first four members (D, D, A, D) of the harmonic series played, it will be found that the lower three notes are in quite accurate tune relative to one another, while the fourth is a trifle flat. This may be verified for the upper two notes by comparing them with the same notes played with conventional fingering. If a similar experiment is carried out by use of the fingering for F, only the first three members of the sequence (F, F, C, F) will be found to be in reasonable tune. The high F is, in this experiment, quite flat. One further experiment is needed to complete the series. Only the lower two notes of the harmonic series (B♭, B♭, F) based on the fingering for B♭ will be in good tune, and once again the highest note is flat.

The interpretation of these experiments is as follows: As shown in Fig. 10, the head-joint taper produces a sharpening effect on high notes only up to about 1000 cps for a Boehm flute. Those members of a harmonic series that have their fundamental frequencies below this value will be in tune, because the progressive sharpening produced by the head joint is able to overcome the flattening associated with the regeneration mechanism. Notes that are higher than this limit will not be corrected by the head joint, and so will be flat unless helped by the presence of a vent hole. In all the experiments, nonvented notes higher than C (1050 cps) are flat in an amount that increases with their distance above C.

Figure 11 shows that altering Δl_e by covering more or less of the embouchure hole shifts the frequency at which the head joint ceases to correct the regenerative flattening. Playing the flute as flat as is comfortably possible (so that Δl_e is about 55 mm) shifts this critical frequency down from the C near 1000 cps to the B near 900 cps. At first sight, it would appear that a repetition of experiments similar to those described above would serve to demonstrate this shift. The earlier phenomena apparently would be reproduced if the test notes were all shifted down a semitone so as to relate them correctly to the lowered critical frequency. This transposition would have to be done by pulling out the slide enough that the flattening produced by this means plus that produced by the increase in Δl_e would lower the frequency of the notes the desired amount. Experiment does indeed verify the lowered frequency at which the head-joint compensation ceases, but it proves not to be meaningful to use the normal vented fingerings as a way of producing the reference frequencies. Lowering the frequency by pulling the slide and covering the embouchure hole has the effect of placing the vents relatively closer to the nodal positions than they are in normal use. As a result, the reference notes are themselves flat because of undercompensation, thus reducing the apparent discrepancy between the vented and unvented notes. Furthermore, under the conditions of this experiment Δl_e itself would decrease as the frequency rises, and so would sharpen the upper note relative to the low ones regardless of the fingering that is used to play them.

V. COMPARISONS OF DIFFERENT BORE DESIGNS

The Boehm design of bore, having a tapered head joint and cylindrical body, dominates contemporary practice in the making of flutes; however, the older design, which uses a cylindrical head on a tapered bore, still persists in the modern piccolo. Completely cylindrical flutes and fifes are also possible and are in common use in many parts of the world. Because of the variation among the acoustical properties of these three

[29] Already in the lower two registers, the use of finite-sized tone holes gives rise to an end correction that requires them to be placed closer by amount Δl_0 to the embouchure hole than the position of the nominal open pipe end belonging to the note in question; see Ref. 8, Sec. V-C. The presence of *closed* finger holes above the vent requires a shifting of the hole position by a further amount Δl_c that is given approximately by Eq. (7) of Ref. 8. The numerator of this equation should read $r[1-(\bar{v}/c)]$ and not $r-[1-(\bar{v}_c/c)]$.

[30] Reference 4(a), Vol. 2, p. 137.

designs, it is of considerable interest to compare them with one another, with the Boehm pattern as a standard.

A. Cylindrical-Head, Conical-Bore Flute

As has already been shown in Figs. 8 and 10, a cylindrical head on a conical flute provides a contraction of the original bore of a sort that flattens the lower playing frequencies more than the higher ones. Standard practice for flutes of this sort (Quantz' instrument) gives a fractional change (S_p/S_0) in cross section at the embouchure hole of about 0.21 as against 0.16 for the Boehm flute (see Fig. 9). As a result, the stretching of the first-octave interval is $(0.21/0.16)^2 = 1.7$ times larger than that of the Boehm flute, as has already been pointed out in Sec. IV-C. The question immediately arises as to whether such a design should not lead to an excessively wide first octave. The answer is in the negative for flutes using the small finger holes that were a normal part of the old design. If, however, the flute is pulled apart at the middle joint, so that the tube is terminated in an open end instead of a row of finger holes, the octave fingered as G, G, does become quite wide.

For frequencies above 1000 cps, the head joint by itself would permit a relatively flat third register. However, the high position of the small finger holes places the third-register vent holes far above the pressure nodes that they are supposed to encourage. Equation (14) shows that these third-register notes would then be pulled very high. The "rational" scheme for venting, which calls for opening the fifth hole above the note to be played (as on the Boehm flute), gives notes that are hopelessly sharp on most old-style instruments, so that hybrid fingerings are normally used.

The foregoing discussion implies the possibility of designing a well-tuned, rationally vented flute by use of the Quantz type of bore, simply by enlarging the tone holes enough that they can be moved down to the point where they sharpen the third-register notes only the correct amounts. It would appear that the conical-bore Boehm-system flute[31] of 1840 was designed to meet these requirements.

Another feature of the cylindrical head used with a conical body is worth mentioning: Figure 8 makes it clear that pulling out the head-joint slide not only increases the perturbed length of the bore but also the magnitude of (S_p/S_0), causing a further steepening of the slope and at the same time an additional flattening of the low notes beyond the amount expected from a simple lengthening of the instrument. The increased steepness of the Δl_t curve gives rise to an octave-stretching that grows rapidly as the slide is pulled out. On any such flute, there is a slide position that gives the best octaves in practice, a position that may not coincide with the positions that give optimum *consistency* of intonation or best over-all absolute frequency. A workable compromise is possible in many cases, however, since the cork can also be moved to adjust the intonation. Figure 3 shows that moving the cork a few millimeters either way produces a negligible change in tuning at the bottom of the flute's range (near 300 cps). The higher notes can be altered quite considerably through the direct effect of Δl_c (Fig. 3) and through its indirect effect on Δl_t (Fig. 11), as explained in Sec. IV-C. These two alterations act on the frequency in opposite directions, with the indirect one dominant in the lower two registers (below 1000 cps) while the direct one dominates in the third register. Thus, the slide should be used to tune a low note to the desired absolute frequency, after which the cork provides a means for adjusting[32] a high note.

B. Completely Cylindrical Flute

The completely cylindrical flute must meet design criteria that are somewhat different from those for instruments possessing a constricted head joint. The contracted octave relation between the lower two registers (produced by the edgetone mechanism and possibly by small finger holes) must now be fully counteracted by a progressively decreasing magnitude of the embouchure length correction Δl_c. As has already been pointed out, the Boehm design of flute has a very nearly constant value near 42 mm for Δl_c and a taper correction that decreases with rising frequency at the rate of about 2 mm/100 cps (over the range of the lower two registers). A similar sharpening may be obtained from the embouchure correction alone, if the reference Helmholtz resonator frequency f_{17} is lowered slightly below the normal for cylindrical-head conical flutes to about 1300 cps, and if the cork is also moved as close as possible to embouchure hole (see Fig. 3). The low value of f_{17} can easily be achieved if one makes the embouchure hole a few percent smaller than the normal size, by using a higher riser, by undercutting it less, or by covering a normal-sized hole further than usual with the lips.

The foregoing choice of the embouchure-hole dimensions and cork position will give satisfactory intonation in the two lower registers, but the third register will be somewhat sharp if the Boehm scheme of venting is used (unless the holes are extremely large). However, the large value of Δl_c makes this type of instrument considerably more sensitive than Boehm's to the player's lip position, so that in practice it is not difficult to control the tuning.

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[31] Ref. 1, Chap. 1, Fig. 6. The instrument is No. 409 in the *Dayton C. Miller Collection Checklist* compiled by Laura E. Gilliam and William Lichtenwanger of the Library of Congress, Washington, D. C.

[32] Reference 3, p. 156, gives a rule of thumb that approximates this procedure. He does not state explicitly that the procedure is properly applied only to the conical flutes with cylindrical heads. The Boehm flute does not have its basic octave intervals upset by ordinary shiftings of the slide.

VI. RECAPITULATION

The results of the present investigation into the acoustical nature of the flute head joint are recapitulated here to give a succinct summary of a somewhat extended account.

It has been shown that to a good approximation the endcorrection arising at the upper end of the flute owing to the cork-to embouchure cavity is very nearly constant at 42 mm in the normal playing on the Boehm flute. Resonances in the player's mouth cavity produce a sharpening of a few percent of a semitone, but only at frequencies above a critical value near the resonance frequency.

It is inherent in the nature of the edgetone regeneration mechanism that produces the flute tone that the upper modes are excited at a frequency that is flat relative to the lower ones. Analysis shows that it is the function of the contracted bore in the head joint to correct this discrepancy over the first two octaves of the flute's range. This contraction can be either as a conoidal contraction at the upper end of the flute (as is the case in Boehm's design) or as a cylindrical section that serves as an effective contraction of an otherwise conical bore (as in the case of the older style of flute). The work of correction in the third octave of the flute's range is given to the vent holes used in fingering these notes. The nature of the requirements on hole size laid down by the necessity for this correction has been described briefly.

Finally, the analytical results described above are applied in the course of a description of the structure and musical properties of the two major styles of flute-bore design that were referred to in the preceding paragraph. It is also shown that under certain special circumstances a completely cylindrical bore can be made to give acceptable intonation.

ACKNOWLEDGMENTS

The authors are indebted to many persons for help of various kinds. The major contributors and their contributions are listed here. John C. Stavash has on several occasions presented his professional viewpoint in illuminating discussion of the playing, construction, and intonation of flutes. Walter Mayhall has also been a ready source of information from the musician's point of view. The opportunity to review the mathematical analysis on many occasions with Robert H. Scanlan has been most helpful because of his familiarity with both the acoustical and musical aspects of the problem. We are also grateful to William Lichtenwanger of the Library of Congress for an opportunity to examine and measure a number of instruments belonging to the Dayton C. Miller Collection. James T. Bennett performed several extremely tedious numerical integrations of the head-joint-taper perturbation at an early stage of the development of this report. Bruce M. Shoffner more recently programmed and carried out a number of more extensive calculations at the Case Institute Computing Center. Some of these calculations are summarized in Figs. 2, 3, 10, and 11. He also made available a Boehm-type piccolo for a number of experiments.

The following list gives the names of various individuals who were kind enought to permit us to make physical measurements of flutes in their possession. Because of its possible relevance to the scientific side of this report, the serial number and maker's name is given for each instrument.

Matthew Jones: Powell No. 9; W. Haynes Nos. 14362 and 16041, as well as an unnumbered W. Haynes head joint; Rudall Carte No. 3900; Boehm and Mendler (no number).

John C. Stavash: Powell Nos. 1626 and 1993; Gemeinhardt No. 3755; Reynolds No. 38116.

William Lichtenwanger, *Library of Congress:* Boehm and Greve (8 key) Checklist No. 240; Boehm and Mendler (Boehm system) Checklist Nos. 134 and 233; also, the alto flute Checklist No. 305, by the same makers

Earle L. Kent, *C. G. Conn Ltd.:* Artley, serial number not recorded (1962).

J. F. Feddersen, *H. and S. Selmer Company:* Signet No. 12769; Bundy No. 72583.

In addition to these instruments, the following instruments belonging to one of us (AHB) were also studied: G. Haynes No. 3904; also, a considerably older one without serial number by the same maker. Reynolds No. 26704; Laube (6-key), without serial number; Gemeinhardt piccolo No. 2512, and an unmarked 6-key wood piccolo that formerly belonged to P. H. Benade.