Received 16 May 1968 6.8

Sounding Mechanism of the Flute and Organ Pipe

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Measurements on an artificially blown and mechanically excited flute head joint provide values of the complex acoustic back pressure generated by the blowing jet. The magnitude of the acoustic back pressure is calculable from the jet momentum and is approximately twice the static blowing pressure times the ratio of the lip-aperture area to the tube cross-section area. The phase of the induced back pressure relative to the oscillation volume velocity is determined by the lip-to-edge distance and the velocity of propagation of a wave on the jet. Adjustment of this phase is demonstrated to be the major means by which the flutist selects the desired mode of oscillation of the instrument. The efficiency of conversion from jet power to acoustic oscillation power is low (2.4% at 440 Hz) and is about equal to the ratio of particle velocities in the air column and the jet. Nonlinear (turbulent) losses are measured and are substantial. Stroboscopic views of the jet motion under explicitly stated oscillation conditions show the large amplitude of the jet wave and its phase relative to the stimulating acoustic disturbance.

INTRODUCTION

QUALITATIVE theories of the means by which acoustic oscillations are maintained in flutelike instruments have been available at least since that proposed by Sir John Herschel in 1830.[1] The intervening period has seen a certain amount of dissension as to the nature of the mechanism, accompanied by only a few controlled observations. Carrière[2] injected steam into the air jet of a very large organ pipe and observed stroboscopically the vortices formed in this stream. Brown[3] observed in detail the instabilities of a jet of air subjected to an acoustic disturbance, and Sato[4] has recently treated theoretically the mechanics of such a fluid stream. Cremer and Ising[5] treat the self-excited organ pipe as a resonant system coupled by a feedback mechanism to an oscillating jet.

The picture that is presented is briefly this: A thin flat jet of air, subjected to an alternating disturbance near its point of issuance, will develop a sinuousity in the form of a growing wave whose propagation velocity is roughly one-third to one-half the original jet velocity. The disturbances will eventually grow into a series of vortices. In the flute or organ, however, an edge or wedge upon which the jet plays interrupts the jet before these vortices are fully developed, and the result is to provide on each side of the wedge a set of air pulsations at the frequency of the initial disturbance. These pulsations can maintain acoustic oscillations in a resonator to which the wedge is properly affixed, and these oscillations in turn provide the initial disturbance for the jet. Subject to certain phase and loop-gain conditions, the entire system will then maintain itself in oscillation. In general, there are several modes of oscillation that can take place, both with respect to the number of acoustic wavelengths contained in the resonant pipe, and the number of undulant wavelengths of the jet stream, giving rise to a two-dimensional set of possible steady-state conditions that has been described (not entirely correctly) by Benade and French[6] and Bouasse.[7]

The present work inquires quantitatively into the processes involved in converting the direct current of the performer's breath into the alternating oscillations of the acoustic resonator, and how the oscillations depend on the parameters of the blowing mechanism. While the investigation has been limited to a single geometry and a relatively small range of frequencies, it has provided enough information to formulate a simple quantitative theory that appears adequate to

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[1] R. S. Rockstro, *The Flute* (Rudall, Carte & Co., London, 1928), p. 34.

[2] M. Z. Carrière, J. Phys. 2, 53-64 (1925).

[3] G. B. Brown, Proc. Phys. Soc. (London) 47, 703-732 (1935).

[4] H. Sato, J. Fluid Mech. 7, 53-80 (1960).

[5] L. Cremer and H. Ising, Acustica 19, 143-153 (1963).

[6] A. H. Benade and J. W. French, J. Acoust. Soc. Amer. 37, 679-691 (1965).

[7] H. Bouasse, *Tuyaux et Resonateurs* (Librarie Delagrave, Paris, 1929).

explain many aspects of the behavior of the flute and organ pipe with reasonable accuracy.

I. NATURE OF THE JET MOTION

Carrière's pictures[1] of jet streams were taken with an enormous (50-ft) organ pipe whose proportions were quite different from those of the flute. Moreover, he reported no measurements of the strength of oscillation. It seemed desirable then to obtain some visual information about the jet motion under conditions characteristic of the flute. Accordingly, a flute head joint, provided with acoustical driving and measuring mechanisms to be described later, was blown with an artificial air supply through an orifice closely resembling that of the flute player's lips. A small stream of cigarette smoke introduced into the jet, and a phase-locked stroboscope, permitted observation of the jet form. In Fig. 1 are sketches of the observed form of the jet. Each picture is for a specific phase relative to the acoustic current (volume velocity) at the mouthhole, as marked. A caution is raised about these and similar pictures: They are merely snapshots of the configuration of the smoke at a given instant. It must not be inferred that the smoke particles follow paths such as these in getting to that position, nor should it be assumed that air that does not contain smoke is not also in motion.

The conditions for these pictures are typical of the flute when playing A at 440 Hz moderately loud. Some features of the motion are worthy of comment. *First*, it can be seen that the jet does not instantly alter its form in response to the acoustic current. At 90°, the acoustic current has been moving out of the mouthhole during $\frac{1}{4}$ of a cycle, but the jet is just now blowing over the edge rather than into the hole. *Second*, the jet reacts strongly to moderate disturbances. Its own initial velocity in this case was 1320 cm/sec. The acoustic particle velocity (which acts roughly at right angles to the jet) was about 300 cm/sec. The acoustic particle amplitude of motion is about 1 mm; the jet, however, moves laterally at least 10 times this distance. A quite complete switching action thus occurs as opposed to the modulation that would obtain if the lateral jet motion were comparable to the jet thickness.

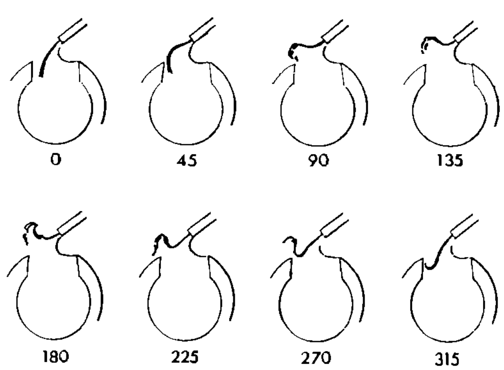

Fig. 1. Sketches of smoke-laden jet viewed stroboscopically. The labels are phase angles of the acoustic current (volume velocity) at the hole; 0° is zero current, 90° maximum current blowing out. Frequency 437 Hz, blowing pressure 0.5 in. of water, acoustic volume velocity 130 cm^3/sec.

The experiments to be described were directed at measuring the acoustic pressure engendered in the resonator by such a switched jet.

II. APPROACH

Much of the previous work on organ pipes has been complicated by the fact that the oscillating system is only weakly under the control of the experimenter—the feedback mechanisms at work permit it to take up a state of oscillation appropriate to the imposed external conditions, and as these are varied, the oscillator alters its frequency and amplitude and may jump hysteretically from one mode to another. In the present work, the feedback loop was disconnected. The ability of the blown embouchure to convert the direct current of the air stream into an alternating acoustic pressure was measured under conditions where the stimulating acoustic vibrations were separately produced at a known amplitude by an electrically driven piston. The embouchure was treated then as a two-terminal impedance connected in series with the equivalent transmission line at the plane of the mouthhole, and whose value was a function of the blowing conditions and of the acoustic current flowing through it. This impedance is complex, and in the region of interest has a negative real component. When the magnitude of this component is larger than the positive resistance of the resonator, oscillation can take place. It will be maintained at an amplitude and frequency determined by the condition that the impedance looking into the embouchure is the negative of the impedance looking into the resonator. The latter was determined with the same apparatus, and also turns out to be nonlinear.

The nonlinear nature of both these impedances has two implications. First, a certain degree of harmonic generation is encountered—i.e., an impressed sinusoidal current gives rise to a nonsinusoidal pressure. Fortunately, the harmonic content in the sound of the flute is not large, and since the radiated power goes up as the frequency squared, we find the harmonic content of the oscillation within the tube is quite moderate. Oscilloscope observation of the sound pressure at the stopped end of the artificially blown head rarely showed harmonic content more than 20% in amplitude. For the purposes of this experiment, the harmonic generation was ignored and only the fundamental pressures measured. This means that the theory presented does not deal with an important aspect of a musical instrument, the tone quality. The second aspect of the nonlinearity is that, since impedances change with amplitude, each measurement must be carried out at some specified oscillation amplitude. We shall find, in fact, that the blown embouchure acts more nearly like a constant-pressure generator than like a constant negative resistance. The impedance concept is, however, a convenient method of expressing the results. It is employed here with the caution that the word *impedance* is used merely to express the ratio of the fundamental of the pressure generated to an impressed sinusoidal volume velocity of a given value.

III. APPARATUS

Figure 2 shows schematically the arrangement used to measure the impedances of the flute sections, and of the jet. It consists of a short length of copper pipe of $\frac{3}{4}$ in. i.d., to which sections like the cylindrical flute head (1) could be affixed. It is closed at one end by a piston (3), whose mass (18 g) is large as compared to that of the air in the tube. This piston is sealed with a thin rubber diaphragm, and may be driven by the loudspeaker motor (4) to provide a variable driving acoustic current. The value of this current could be measured with a pickup coil (5), which moves with the piston in a separate magnetic yoke. Closely adjacent to the closed end is a rigid microphone (6), made from two thin disks of oppositely polarized barium titanate. The microphone, which was calibrated in another laboratory, measures the acoustic pressure close to the closed end. The ratio of the microphone signal to the pickup coil signal is proportional to the acoustic impedance looking up the pipe. This impedance could always be made real by tuning the system to resonance, and it was thus possible to provide a null-balancing circuit in the form of potentiometer (8), from which the acoustic resistance at resonance could be read directly. The detector took the form of an oscilloscope whose x axis was driven sinusoidally by the audio oscillator that drove the loudspeaker. The resultant Lissajous figure showed the presence of nonlinearities, and permitted visual balancing of the fundamental to zero, even with harmonics present.

In order to damp the resonator so that it would not oscillate under the action of the air jet alone, an acoustic resistor (7) was provided. In an attempt to make this resistor noninductive, a bundle (more accurately, a disk) of several thousand glass capillaries, each 0.01 cm in diameter and 0.1 cm long, was used. While the length and diameter of each of these tubes were such as to make its resistance outweigh its inductance by a large factor at all frequencies of interest, the assemblage of tubes taken as a whole had an end correction of the order of the diameter of the entire disk, so that an appreciable inductive effect was measured. The assemblage of tubes could be partially covered by a rubber pad to vary the acoustic resistance. This resistor was used as a test object for calibrating the null circuit. With a quarter-wavelength of open pipe connected and the resistor partly opened, the potentiometer reading at resonance was obtained. The Q of the resonator was then measured by running a frequency-response curve. To avoid problems from acoustic nonlinearities, the driving current was adjusted at each frequency to give a constant pressure, rather than employing the usual technique of keeping the drive constant and measuring the response. Because the loudspeaker motor was driving a mass whose amplitude of vibration would fall off with frequency for a constant driving force, the electrical oscillator was coupled to the amplifier with a small capacitor to give a voltage rising with frequency in compensation.

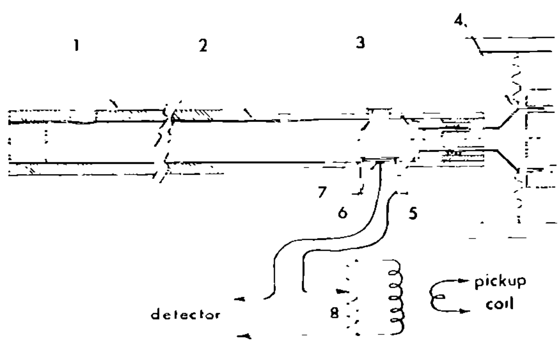

FIG. 2. Apparatus for measuring acoustic impedance: (1) flute head with mouthhole, (2) tuning slide, (3) piston, (4) loudspeaker motor, (5) velocity pickup coil, (6) microphone, (7) acoustic resistor, (8) null potentiometer.

The effective resistance R as seen at the closed end of a resonant length of tube is related to the Q of the resonator by

$$R = 4QZ_0/n\pi. \tag{1}$$

Here Q is the quality factor, n the number of quarter-wavelengths on the line, and Z_0 the characteristic impedance of the tube:

$$Z_0 = \rho c/S, \tag{2}$$

where ρ is the density of air, c the velocity of sound, and S the cross-sectional area of the tube.

The potentiometer was found to give readings directly proportional to the effective resistance, independent of frequency, as it should. Readings were repeatable, the resistance read on successive balances rarely varying as much as 1%.

The procedure for making a measurement of the jet impedance was as follows. With the acoustic resistor capped (i.e., not in the circuit), the jet blowing tube geometry and blowing pressure were adjusted as desired; for example, to produce the loudest possible tone for some chosen blowing pressure. The microphone could be used to measure the acoustic pressure at the velocity node. The acoustic current at the mouthhole is found by dividing the microphone pressure by Z_0, and multiplying by $\sin\theta$, where $\theta/2\pi$ is the distance to the mouthhole in wavelengths. The acoustic resistor was then introduced and adjusted until oscillation ceased. With the jet turned off, the driving piston was activated by the electrical oscillator, the frequency tuned to near resonance, and the amplitude adjusted to give some chosen amplitude of acoustic oscillation as measured by the microphone. The potentiometer and frequency were then adjusted to give a null output at the detector and the potentiometer reading was taken as a measure of the resistance seen at the plane of the piston.

This resistance reflects all of the acoustic losses in the system at the particular amplitude of oscillation chosen; the acoustic power loss is given by the square of the microphone pressure divided by this measured resistance. The blowing jet was then turned on at a given blowing pressure, and the measurement repeated. The effect of the jet is to induce an additional acoustic pressure of some unknown phase and amplitude. A null is obtained again by changing the length of the tube at the tuning slide to take care of the reactive component, and balancing again the potentiometer setting. The piston drive must be changed also to return the system to the original amplitude, since some parameters are nonlinear. The change in length of the tuning slide, and the change in potentiometer readings suffice to calculate the effective impedance of the jet. In order to simplify the procedure, a cylindrical rather than a tapered head joint was used.

IV. CALCULATION OF THE JET IMPEDANCE

The electrical circuit analogous to the acoustic system of Fig. 2 is shown in Fig. 3. The flute head tube is represented by a length l of transmission line, terminated at the left by the parallel combination of the mouthhole inductance L_h and the small capacitance C_s of the cavity between the mouthhole and stopper. The effect of the blowing jet is represented by an unknown impedance Z_j, arbitrarily placed in series with the transmission line at the plane of the mouthhole. We do not really know the details of the interaction of the jet with the flow at this point—all that this placing of Z_j signifies is that to get the effective acoustic back pressure sustaining the fundamental of the oscillation we multiply Z_j by the calculated line current at this point.

At the right end, the circuit is driven by a constant current i through the essentially infinite inductance L_d representing the mass of the driving pistion.

The voltmeter V represents the microphone, while the resistor R_a represents the artificial acoustic resistor. The other acoustic losses in the system are not specifically shown in the diagram, but their effect is felt as a real component R_{in} of the impedance Z_{in} looking up the line at the plane of the piston.

The potentiometer, which measures V/i when Z_{in} is real, reads the value of R_{in} shunted by R_a if the artificial resistor is being used.

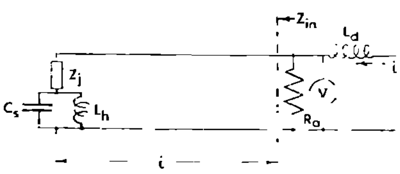

FIG. 3. Transmission-line circuit analogous to the acoustic system of Fig. 2.

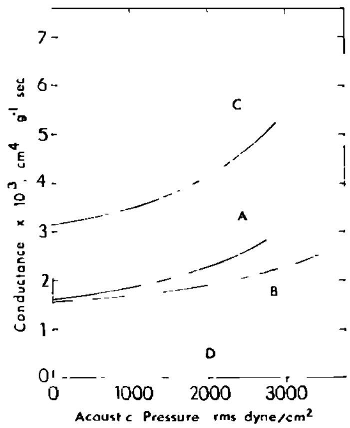

FIG. 4. Acoustic losses for the flute sounding G (392 Hz). The losses are expressed as the conductance as seen at the velocity node, as a function of acoustic pressure at this point. Curve A, head joint alone; Curve B, body alone; Curve C, sum; Curve D, calculated radiation loss.

Calculations of effective impedance are based on the equation for a lossless transmission line:

$$Z_{in}=(Z_L+j\tan\theta)/(1+Z_L\tan\theta). \tag{3}$$

Here, and in the following discussions, all impedances are relative to the characteristic impedance of the line as given by Eq. 2 and are dimensionless. Z_{in} is the impedance as measured looking into a line of length l terminated by a load impedance Z_L, and $\theta=2\pi l/\lambda$, where λ is the wavelength on the line. For the measurement described above, the system is tuned so that Z_{in} is real, i.e., $Z_{in}=R_{in}$. Inverting Eq. 3 and using the condition that $Z_{in}=R_{in}$, we find:

$$Z_L-\frac{R_{in}(1+\tan^2\theta)+j(R_{in}^2-1)\tan\theta}{1+R_{in}^2\tan^2\theta}. \tag{4}$$

In our case, $R_{in}^2\tan^2\theta\gg1$, i.e., the Q of the system without the acoustic resistor is quite high. Making this approximation and taking Z_i as the change in Z_L when the jet is introduced, we find the jet impedance to be

$$Z_j\cong(G_1-G_0)(\cot^2\theta_0+1)+j(1-G_1^2)(\cot\theta_1-\cot\theta_0). \tag{5}$$

G_1 and G_0 are the conductances $(1/R_{in})$ measured by the potentiometer with and without the jet blowing. The angles θ_1 and θ_0 correspond to the lengths of the turned line with and without the jet blowing. Because the acoustic resistor introduced some inductance, a separate measurement was made of the change in line length necessitated by its introduction, and this correction was applied before calculating θ_0 and θ_1. Since only small changes in l are produced by the jet, differential methods were used to evaluate Eq. 5.

V. ACOUSTIC LOSSES

Before describing the jet effects, we report measurements made of losses in flute tubes with the above-described apparatus. These are directly proportional to the conductance $(1/R_{in})$ measured with the potentiometer.

In Fig. 4 are given measurements of the conductance, as seen at the velocity node, of a Haynes flute head

(A) and body (B). The nonlinear effects are apparent in the functional dependence of the conductance on the oscillation amplitude. The flute head shows the most pronounced nonlinearity. The loss coefficient at a typical playing amplitude (2100 dyn/cm² at the velocity node) is about 50% larger than the small-signal value, and rises very rapidly beyond this point. The effects arise almost entirely at the mouthhole, which was partially covered with a modeling-clay "lip" as in normal playing. A cylindrical open-end pipe of the same diameter and resonance frequency shows practically no increase in conductance with amplitude.

Acoustic nonlinearities in small apertures have been treated by Ingard and Ising.[8] They show that the nonlinear effects result from acceleration of masses of air that do not entirely return through the hole on the reverse cycle. The results in Curve A agree with their measurements within 10%, when the mouth hole area (0.63 cm²) is used to calculate the acoustic particle velocity and specific resistance at this point. The flute body shows also some nonlinearity (Curve B, Fig. 4), though it is not as pronounced as for the head. The tone holes are doubtless responsible for this. It is evident that in any quantitative treatment of the flute as an oscillating system, these nonlinear losses must be taken into account. Calculations based on small-signal Q values would not be representative of what goes on at normal playing amplitude.

VI. IMPEDANCE OF THE JET

By use of the technique described above, a number of measurements were made of the effective jet impedance as a function of blowing pressure, distance from the end of the blowing tube to the edge of the embouchure hole, and frequency and amplitude of the exciting acoustic oscillation. An effective way of describing the results is to plot, in the complex plane, the impedance of the jet as a function of blowing pressure, all the other parameters being held constant. Sets of these plots are then made for other values of the fixed parameters.

A typical plot of this kind is shown in Fig. 5. Impedance values are relative to the characteristic impedance of the tube, 15.3 g $cm^{-4} \cdot sec^{-1}$. The sign convention is that appropriate to the impedance as seen from inside the tube, i.e., a positive real value represents a resistive loss, a negative real value, an energy generator, a positive imaginary value, an inductive (mass) loading, and a negative imaginary value, a capacitance or compliance. In the particular measurement reported in Fig. 5, the acoustic pressure p_m at the microphone was held constant at an rms value of 450 dyn cm^{-2}. The calculated acoustic volume velocity at the plane of the mouthhole, a distance $l=\lambda\theta/2\pi$ away, is given by $(p_m/Z_0)\sin\theta$. For Fig. 5, its value was 28 cm³ sec^{-1} rms. These values, about $\frac{1}{4}$ those for loud playing, were chosen for presentation here because the resulting diagram exhibits all of the essential features of the cases examined. Each point on the curve corresponds to a particular jet blowing pressure, measured and labeled in inches of water. The geometry of the blowing tube was held fixed throughout this set of measurements.

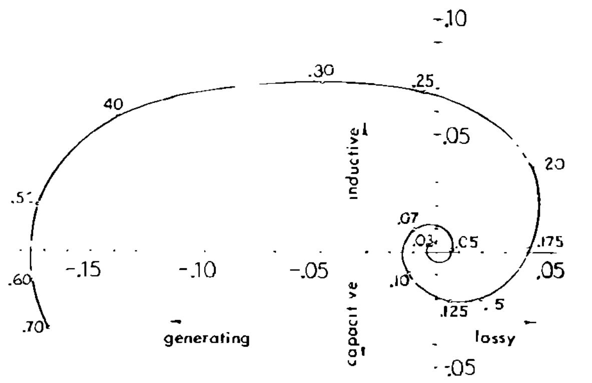

Fig. 5. Complex acoustic impedance of the jet as seen from inside the tube at the mouthhole. Impedances are relative to the characteristic impedance of the tube, 15.3 g $cm^{-4} \cdot sec^{-1}$. Labels on the points are blowing pressure in inches of water. Acoustic current at the mouthhole constant at 28 cm³ sec^{-1} rms. Frequency: 440 Hz. Jet orifice area: 0.072 cm².

It can be seen that the impedance ascribable to the jet is a smooth, well-behaved function of blowing pressure over the entire range, the magnitude decreasing monotonically as the pressure is reduced, and the phase rotating clockwise over more than two complete cycles. The impedance values can lie in any quadrant of the complex plane. Starting at the outer edge where the blowing pressure is about 0.56 in. of water, we see that the impedance is real and negative; such a condition would overcome the losses in the tube were the artificial resistance removed, and would result in oscillation at large amplitude at the natural resonance frequency of the tube. At a higher blowing pressure, the impedance has a capacitive component that will make the frequency sharp: at lower pressure, 0.3 in., the inductive effect makes it go flat, and the real component is less negative—i.e., it could not generate so much power. At about 0.25 in., the phase crosses into the positive real domain—the jet now represents a loss mechanism and could not possibly sound the flute. At about 0.11 in., however, we are back into the negative resistance region, which persists down to about 0.06 in., where the curve crosses over again into the positive domain.

It is evident that while the jet impedance varies smoothly with blowing pressure, in principle spiraling indefinitely around the origin as the pressure is reduced, it represents a possible sound-generating mechanism only when it lies in the negative half-plane, which it periodically occupies as the blowing pressure is reduced. In traversing the negative resistance region, the impedance crosses the real axis, going from capacitive to inductive reactance. In the flute under test, oscillation at 440 Hz was obtainable for only two such pressure régimes, separated as expected by a zone of silence,

[8] U. Ingard and H. Ising, J. Acoust. Soc. Amer. 12, 6 17 (1967).

TABLE I. Propagation velocity of a disturbance of the jet. Frequency, 440 Hz. Transverse sound field particle velocity: 28 cm/sec rms; peak to peak displacement amplitude: 0.028 cm.

Blowing pressure (in. water)	Initial jet velocity u (cm/sec)	Phase velocity of disturbance (cm/sec)	Velocity ratio
1.0	1920	670	0.35
0.6	1460	600	0.41
0.3	1000	390	0.39
0.15	770	370	0.49

with the generated frequency going from sharp to flat with lowering pressure with each régime. In organ pipes, oscillation at the same frequency for several distinct pressures may be observed.[6,7] While in principle there are an indefinite number of turns as the pressure is lowered, there is a last turn for high pressures. This is because the phase of the impedance depends on the travel time of a wave of the jet across the mouthhole. As the velocity decreases, the travel time can become indefinitely long, encompassing an arbitrary number of cycles before reaching the splitting edge. With increasing velocity, the travel time can only approach zero. This last turn is the large one to the left. It terminates at high blowing pressures (in Fig. 5 at 0.7 in. of water) with the onset of a noisy turbulence. Only this major turn of the curve is used in music, and we devote most of our attention to an examination of its properties.

VII. PHASE OF THE JET IMPEDANCE

The rotation of the impedance vector with pressure is associated with the travel time of a jet disturbance across the mouthhole. If we plot the phase of the points in Fig. 5 against the inverse of the initial airstream velocity u, an essentially straight line results. The slope of this line, together with the known distance across the hole, gives a phase velocity—i.e., the velocity of a disturbance on the jet, about 0.4 the initial jet velocity u. To check this inference, an experiment was conducted with a jet stream from the same blowing tube injected into the strong transverse sound field existing between

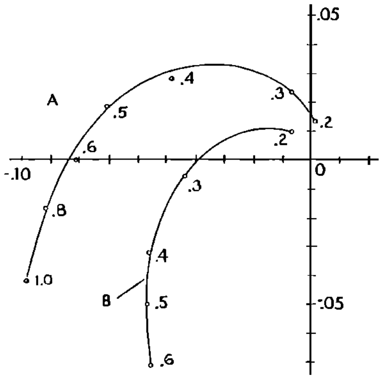

FIG. 7. Acoustic impedance of the jet for two lip-to-edge distances: Curve A for 7 mm and Curve B for 5 mm. Labels as in Fig. 5. Acoustic current at the mouthhole 74 $cm^3 sec^{-1}$ rms. Frequency 440 Hz. Jet orifice area 0.072 cm^2.

two opposed quarter-wave resonant pipes driven by the oscillating piston. Movies of the stroboscopically illuminated, smoke-laden jet were examined frame-by frame to measure the propagation velocity of the disturbances on the jet. Typical frames are shown in Fig. 6. The measured velocities were remarkably constant along the path; there was no evidence of any slowing down on the propagation velocity right up to the point where the smoke trace broke up. Table I lists the values found.

The values found check well those inferred from the rotation of the impedance vector. Experiments measuring rotation of the impedance vector as the lip-to-edge distance is changed give closely concordant values. Sato[8] and Brown[2] discuss the theoretical and experimental aspects of this wave propagation. Cremer and Ising[5] derive an equation for the expected jet motion and compare it with observations on an organ pipe jet. It suffices to say here that even for the relatively strong disturbances acting in the case of the flute and organ pipe, the wave-propagation velocity stays very close to 0.3 u to 0.4 u, as these authors find.

The phase of the impedance vector, which is so important in determining the strength and frequency deviation of the resultant oscillation, is thus determined by the initial jet velocity—i.e., the blowing pressure, and the lip-to-edge distance. Figure 7 shows two spirals like that of Fig. 5. Curve A was obtained with a 7-mm lip-to-edge distance, Curve B with 5 mm. The effect of changing the distance is to rotate the entire diagram, which gives rise to a pronounced change in the oscillation condition. With a 5-mm distance and 0.6-in. blowing pressure, the jet wave gets there too soon; its capacitive reactance would make the flute sound half a semitone sharp. Reducing the pressure to 0.3 in. to avoid this would give a weak oscillation. By pulling the lip back to 7 mm, however (Curve A), the arrival time is delayed, and a strong in-tune oscillation could be produced at 0.6-in. blowing pressure. There is no pressure on Curve B that could match this oscillation strength.

The necessity for adjusting the lip-to-edge distance is further brought out by examining the effect of frequency change. Figure 8 shows two impedance plots

FIG. 6. Snapshots of jet in sound field. Four frames ¼ cycle apart in time. Blowing pressure 1.0 in., initial jet velocity 1920 cm sec^{-1}. Acoustic peak particle velocity 38 cm/sec.

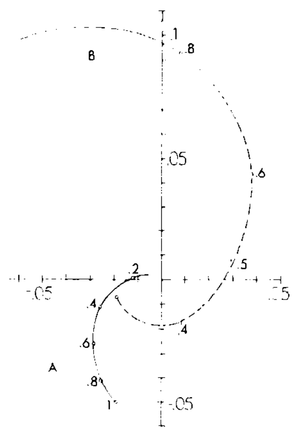

FIG. 8. Acoustic impedance of the jet for two modes of the resonant pipe. Curve A: first mode, 300 Hz, acoustic current 135 $cm^3\ sec^{-1}$. Curve B: second mode, 900 Hz, acoustic current 82 $cm^3\ sec^{-1}$.

taken for identical blowing geometries and pressures; Curve A for the 300-Hz first mode of a stopped pipe, and Curve B for its second mode, at 900 Hz. It can be seen that the phases are markedly different. At a blowing pressure of 0.6 in. of water, the jet impedance vectors for the two modes are about 180° apart. At this pressure, the jet impedance for the second mode (900 Hz) lies in the real half-plane—it could not produce oscillation at this frequency. As the pressure is increased, the impedance vector rotates counterclockwise (note that its rotation rate is three times faster for 900 Hz than for 300 Hz, as expected) so that at a blowing pressure of 1.0 in., it lies well in the generating quadrant, and the upper mode would be sounded. Transition to the upper mode could be greatly favored by moving the lip closer to the edge; this would rotate both diagrams counterclockwise so as to put the upper-mode vector in the generating region over most of the curve, and displace the lower-frequency mode toward the lossy region. Conversely, retraction of the lip would place the low-frequency mode in a favored position, and disadvantage the other. Curves run for the conditions of Fig. 8, but with a lip-to-edge distance of 9 mm instead of 7 mm, show that exactly this happens, Curve B being rotated completely into the nongenerating half-plane, while Curve A rotates one-third as much toward the generating axis.

This adjustment of the lip-to-edge distance by the flute player, and its effect on intonation and tone production have been discussed by Coltman.[9] It is apparent from the above that the flute player adjusts both the blowing pressure and lip-to-edge distance in such a manner as to control the arrival phase of the jet, and that this phase is a more important variable in determining which mode will be sounded than is the magnitude of the blowing pressure.

VIII. ACOUSTIC PRESSURE GENERATED BY THE JET

It is important to point out that the magnitudes of the measured jet impedances such as shown in Figs. 5, 7, and 8 depend markedly on the value chosen for the magnitude of oscillation. If the jet impedance is multiplied by the current at the plane of the mouthhole, one gets the complex acoustic backpressure generated by the jet, which may be plotted in a similar diagram. Such curves are much less dependent on the value of the current chosen, and we conclude from this that the jet action can be best described in terms of the magnitude and phase of the acoustic pressure that it generates. The way in which the phase varies has already been discussed; we give attention now to the mechanisms that determine the acoustic driving force that the jet can provide.

In the following discussion, we neglect certain refinements in quantitatively dealing with flow—namely, we presume that viscous and friction effects at the walls are absent. This is a reasonable approximation for the precision sought here.

A jet of air issuing from an orifice of area s_1 under the influence of a blowing pressure p, will have an initial velocity u given by Bernoulli's law:

$$p=\rho u^2/2. \tag{6}$$

It will carry a volume of air us_1 per second. Consider such a jet blowing axially into the open end of a long tube of larger cross-section area s_2 whose far end is closed. The jet stream will mingle with the still air, slowing down not only to zero, but in fact reversing direction and re-emerging from the open end with a velocity $-us_1/s_2$. The mass flow is ρus_1, the velocity change is $u+us_1/s_2$, and the total force exerted on the large tube is thus $\rho u^2 s_1(1+s_1/s_2)$. Dividing by the area of the large tube, and making use of Eq. 6, we find the pressure in the large tube is

$$p_2=2p(s_1/s_2)(1+s_1/s_2). \tag{7}$$

Experiments of this sort, carried out with jets similar to those used to blow the flute, bear out Eq. 7 for a variety of pressures and tube areas. The pressure built up in the large tube is independent of small changes in the direction and position of the jet, whether or not it is playing against the wall of the tube, thus justifying the neglect of wall friction. Because the acoustic particle velocity in the flute is small as compared to the jet velocity, and because the jet stream slows down in a distance short as compared to the wavelength, the situation in the flute during each half-cycle is quite comparable to the static incompressible situation described above.

While the force available from the jet is thus known from the rate of momentum transfer, the pressure which this force will develop depends on the cross-section area of the region in which the jet slows down. In the flute, this region is ill defined; the jet acts partly in the mouthhole, whose uncovered area may be 0.5 cm^2, and partly in the tube, whose area is about 2.5 cm^2. To examine situations of this sort experimentally, the

[9] J. W. Coltman, J. Acoust. Soc. Amer. 40, 99–107 (1966).

arrangement sketched in Fig. 9 was made. A long tube of 1.9 cm i.d. was provided with necks of varying length having a diameter of about 0.8 cm. The shortest "neck" was simply a hole in a thin metal end plate. The pressures built up in the large tube by a jet of 0.315-cm diameter, carrying 338 dyn of thrust, are plotted in Fig. 9 as functions of the length of neck. The three curves are for various spacings of the nozzle, including a case where the nozzle extends 4 mm inside the tube. Broken Line A is calculated by Eq. 7 for a long tube of the small diameter. It is seen that when the neck is 3 cm or longer, all the important action seems to be taking place in the small tube. For shorter necks, the pressures drop drastically, but do not reach, even for "zero" length, the low pressure (Line B) calculated for the large-diameter tube with a back-flow velocity dictated by the small-area aperture. It thus appears that an aperture in a thin plate still has an effective length. Or put another way, there is a transition region in the neighborhood of the hole in which velocities of motion are changing from that characteristic of the large diameter to that of the small diameter, and momentum transferred here can result in larger pressures that are transmitted uniformly throughout the volume. In the flute then, we can expect an acoustic pressure to be developed that lies somewhere between that calculated using the area of the tube, and that using the area of the mouthhole.

The smoke traces show that the transverse jet motion is large, and it seems reasonable to consider the jet to be blowing into the tube for a complete half-cycle, transferring its momentum all during this time. It interacts with a sinusoidal acoustic current of some unknown phase ϕ with respect to the square wave of

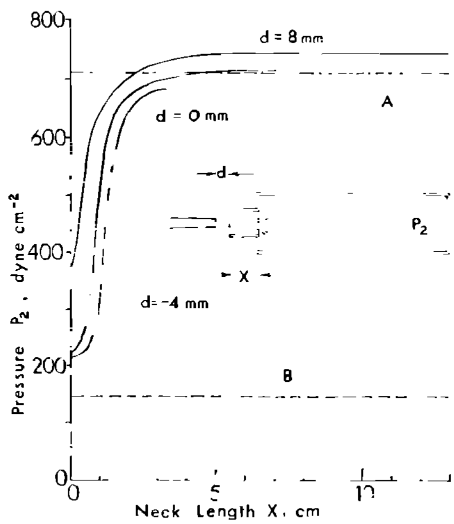

FIG. 9. Pressure produced by a jet blowing into the cylindrical can as a function of neck length. Jet pressure 1.0 cm water, average initial velocity 1944 cm sec^{-1}, flow 145 cm^3 sec^{-1}. Curve A, pressure calculated for infinite neck length. Curve B, pressure calculated for zero neck length. Jet spacings d as marked.

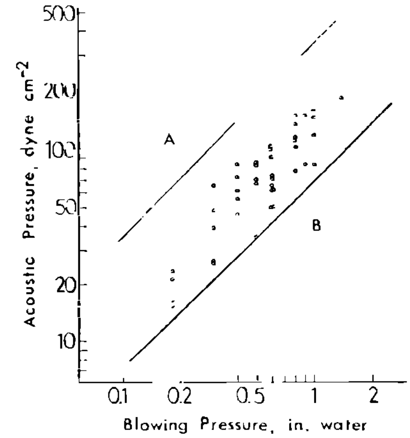

FIG. 10. Acoustic oscillation pressures as a function of blowing pressure for a variety of blowing conditions and frequencies. Curve A: pressure calculated from mouthhole area. Curve B: pressure calculated from flute-tube area.

pressure, and the effective rms value of such a square wave of pressure, with regard to power delivered to the fundamental, will be $\sqrt{2}/\pi$ times the maximum pressure p_2. Using Eq. 7, we can write the magnitude of the rms acoustic pressure generated by the jet as:

$$p_j=\frac{2\sqrt{2}}{\pi}p\left(\frac{s_1}{s_2}\right)\left(1+\frac{s_1}{s_2}\right). \quad (8)$$

Here p is the blowing pressure, s_1 the area of the lip aperture, and s_2 an effective area that lies somewhere between that of the flute-tube cross section and that of the embouchure hole.

Equation 8 is plotted as the two straight lines in Fig. 10, with s_1 the blowing-tube area. The upper line represents a choice for s_2 of 0.5 cm^2 (the embouchure-hole area) and the lower line an area of 2.81 cm^2, the flute-tube area. The points between are the actual observed values of rms acoustic pressure generated. These are obtained from the jet-impedance experiments described earlier and represent values obtained using a wide variety of lip spacings, oscillation pressures, and operating frequencies. All these fall between the lower and upper bounds given by Eq. 8; a mean line through the set lies about a factor of 2 above the lower bound, a value consistent with the findings displayed in Fig. 9 for a short neck. Measurement of the static pressure built up in a flute head by the blowing jet directed just below the embouchure edge, gave a value about 1.5 times that calculated from Eq. 7 when the flute-tube diameter was used for s_2, again showing the influence of the restricted mouthhole in raising the pressure. No measurable suction is developed when the jet is directed across the hole and above the embouchure edge.

While exact values undoubtedly depend somewhat on the particular geometry of the mouthhole and the blowing conditions, it seems safe to state that the acoustic pressure generated by the jet will lie in the neighborhood of twice the blowing pressure, times the ratio of the area of the lip aperture to the area of the flute-tube cross section.

IX. SOUND-POWER PRODUCTION

The oscillation pressure in the sounding flute will reach an equilibrium when the jet impedance, as defined earlier, equals the negative of the impedance looking into the flute at the plane of the mouthhole. At the natural passive resonance frequency of the system, this impedance will be real. If the phase of the jet impedance is such that it in turn is real and negative (which requires a specific combination of jet velocity and lip distance), the oscillation will take place at the resonance frequency of the system and will build up in amplitude until the magnitudes of the impedances match. Since the jet provides a nearly constant-pressure system, its apparent impedance falls inversely as the acoustic current rises. At the same time, the nonlinear effects described in Fig. 4 cause the resonator resistance to rise so the two come to a definite equilibrium point. If the acoustic oscillating current at this point is v_0, the acoustic power generated is $p_j v_0$. The power expended in blowing is pus_1. The generating efficiency is the ratio of these two, and using $p_j = 2ps_1/s_2$, we find:

$$\text{GENERATING EFFICIENCY} \cong 2v_0/s_2u. \qquad (9)$$

Now $2v_0/s_2$ is just the sound particle velocity in the region where the jet is interacting. The generating efficiency is thus equal to the ratio of the sound-particle velocity to the jet velocity. It is clear that the energy loss takes place because in slowing down the rapidly moving jet, momentum is conserved, but energy is necessarily lost. The actual value that v_0 attains is set by the losses in the flute tube. Calculations from the acoustic pressure measured by the microphone and the measured jet impedance show the generating efficiency is low—about 2.4 % at A-440. Even if the resonator were completely lossless, however, v_0 could not rise much beyond the point where the acoustic particle velocity in the mouthhole equaled the jet velocity, for at this point there would no longer be anything to push against. We estimate the efficiency at this point at about 4%, so the mechanism of sound generation in the flute is inherently a very inefficient one.

Of the acoustic power generated, only a small fraction is radiated. The radiation resistance of a small isolated source is $\pi\rho c/\lambda^2$. The flute, under most circumstances, has two sources, one at each end. The one at the mouthhole has slightly less current, owing to the taper and the end correction, but is partially baffled by the player's head. Over the first two octaves, it is nearly as strong a source as the open end. The two interfere to some extent; but the effects are not large, amounting to an increase of 20% or so in the radiation resistance. Using these calculated values, and the total losses measured as in Fig. 4, we find that at 440 Hz only 3.3% of the acoustic power generated is radiated as sound. The over-all efficiency at this frequency then comes to 8×10^{-4}, a value lying within the range reported by Bouhuys.[10] The efficiency will rise with frequency because of the increased radiation resistance.

X. FREQUENCY PULLING

When the jet velocity and lip-to-edge distance are not such as to make the jet impedance real, the steady-state condition will be one in which the frequency is shifted to introduce a reactive component equal and opposite to that of the jet. Inspection of the spirals of Figs. 5, 7, and 8 makes it apparent that reactive components as large as the real components—i.e., jet phases 45° away from the negative real axis—may readily occur and still leave enough real component to sustain the losses. Since the real (loss) part of the resonator impedance must equal the negative real component of the jet impedance, the frequency shift in this case will be such as to introduce a reactive component equal to the resistive component of the resonator impedance, or a shift $\Delta f = f/2Q$. We may thus expect frequency shifts of at least this much as blowing conditions are varied. From Fig. 4 and Formula 1, we find the Q at 440 Hz to be about 30, and thus we expect to find frequency shifts of about ±30 cents. This is quite consistent with those measured on an artificially excited flute.[9] We may also infer from Ref. 9 that the flutist ordinarily operates so as to maintain the phase of the jet impedance at 180°.

While the impedance spirals, and the measurements of frequency shift show clearly that a specific arrival phase of the impulse is required for zero frequency shift, they do not tell us what the required phase is. This is because the actual momentum transfer takes place over a distributed region of the tube, and selection of a particular arrival time is arbitrary. Since it is known, however, that a sound field produces its major influence on the jet immediately after the jet leaves the aperture, we might ask what travel times, from the jet to the edge, are required to produce an acoustic pressure in phase with the ingoing (negative) current.

An examination of the data from many experiments shows that the 180° phase condition is obtained when the travel time of a jet particle across the gap is about 0.2 of a period. Remembering that the jet wave travels at about 0.4 the initial stream velocity, this corresponds to a travel time of ½ a period for a jet disturbance. The oval mouthhole geometry did not lend itself to a more precise determination of this number.

XI. DISCUSSION

Cremer and Ising[5] have approached the description of the organ pipe in a manner similar to that of this paper. In their model, they consider the resonator and jet as coupled systems whose transfer functions must combine to unity. The jet is assumed to inject its pulsating current into the resonant system in such a way as to create a driving pressure against the impedance of the

[10] A. Bouhuys, J. Acoust. Soc. Amer. 37, 453–456 (1965).

mouthhole. Such a model leads to a driving pressure directly proportional to the jet volume velocity. To account for their experimental result that the driving pressure was directly proportional to the square of the initial jet velocity (Fig. 10 of Ref. 5), they hypothesized that entrainment of air by the jet increased the injected current over that of the jet alone, and that this entrainment varied with jet velocity in just such a way as to give a square-law result. The momentum concept presented here in Sec. VIII, however, gives directly the square-law dependence, whether or not entrainment takes place. In a current-drive concept, there is an uncertainty as to the proper place to insert the jet current in an equivalent electrical circuit. In the real case, it is distributed over the region conventionally represented by the lumped reactance of the mouthhole mass, and alternately, flows in either direction through a portion of this region. Cremer's assumption that it is injected between the main resonant column and the mouthhole impedance is hard to justify. The quantitative results to date indicate that the main features of the drive mechanism can be accounted for by the jet momentum, and the volume of gas inserted does not play a major role.

Ingard and Ising[8] show that nonlinear acoustic losses in an orifice are affected by a superimposed steady flow. This raises the question of whether part of the measured effect of the jet represents a modification of the turbulent losses, which are certainly present at the mouthhole. There is also a question of the extent to which the puffs of air from the jet passing outside the wedge contribute directly to the radiated sound. The volume velocities of the jet current and the oscillating sound current are quite comparable in many cases. Neither of these questions has been investigated.

As pointed out in Sec. III, the present work was restricted as much as possible to the case of sinusoidal oscillations. While this is helpful in simplifying the physical picture of what is going on, and is not too bad a model for the flute, the organ pipe is ordinarily constructed to be rich in harmonics and to operate with a jet configuration giving short impulses. Elder and Fasnacht[11] have investigated the velocity and pressure conditions at the mouth of a diapason organ pipe, and point out the importance of the complex waveform in influencing the jet. The interaction of complex sinuous waves on the jet (which show some variation of phase velocity with frequency[5]) with the waveform from a resonator whose modes are not exact harmonics, is a subject whose scope is attested to by the variety of tone colors achieved in the pipe organ. Benade and Gans[12] have investigated for reed and brass instruments the effect of inharmonicity of the modes of the resonator on the regeneration, and find that it is greatly facilitated in a horn whose resonance frequencies are close to a harmonic series. Detailed treatment of regeneration in the case of nonsinusoidal oscillations has yet to be attempted.

XII. CONCLUSION

The acoustic driving force represented by the momentum of the blowing jet is converted into an alternating acoustic pressure on the oscillating air column in the immediate neighborhood of the mouthhole. The phase of this alternating pressure with respect to the alternating acoustic current depends on the lip-to-edge distance and the propagation velocity of a wave on the jet, which is approximately 0.4 the initial jet velocity. The magnitude of the driving pressure is directly proportional to the blowing pressure and the area of the lip aperture. In order to maintain the driving force in proper phase relation with the acoustic current, the flutist increases the blowing pressure and decreases the lip-to-edge distance with ascending frequency. He can independently control the amplitude of oscillation by varying the area of the lip aperture. The octave jump can readily be controlled by choosing pressures and lip-to-edge distances such that the phase condition for oscillation in the desired mode is satisfied, while that for the other mode is not. The momentum transfer concept, and recognition of the phase condition for jet arrival, provides a theoretical basis for the description of the sound generating mechanism of acoustic oscillators of the flute family.

[11] S. A. Elder and W. E. Fasnacht, J. Acoust. Soc. Amer. 42, 1217(A) (1967).

[12] A. H. Benade and D. J. Gans, "Sound Production in Wind Instruments," Proc. Conf. Sound Production in Man, New York Acad. Sci. (Nov. 1966).